# Trade Policy Ambiguity

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**Abstract**

This paper studies how Knightian uncertainty about the distribution of future trade policies affects current trade flows using a dynamic trade model with a sunk cost of exporting. Qualitatively, trade-policy ambiguity reduces export participation in a similar manner to standard trade-policy risk, but also increases sensitivity to tariff bounds, decreases sensitivity to the likelihood of a tariff increase, and can either increase or decrease sensitivity to tariff persistence. Quantitatively, ambiguity dampens the response of trade to persistent reforms and strengthens the response to transitory reforms.

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# 1 Introduction

Uncertainty about future trade policy affects current trade flows by discouraging export participation. The trade-policy uncertainty literature has focused on historical episodes featuring well-defined tariff schedules and risk over which schedule exporters will face. In the case of U.S.-China trade during the 20th century, for example, the risk was that the United States would revoke China's Normal Trade Relations status, which would have switched it to the Non-Normal Trade Relations schedule (Pierce and Schott, 2016; Handley and Limão, 2017; Alessandria et al., 2025a). Starting in 2018, however, the United States has imposed tariff increases on geopolitical foes and close allies alike that depart markedly from the tariff schedule codified in U.S. law, increasing the risk of future tariff changes but also uncertainty about what shape those changes could take. This paper studies how trade-policy ambiguity—Knightian uncertainty over future tariff distributions—affects export participation and aggregate trade.

First, I analytically characterize the effects of trade-policy ambiguity in a Handley and Limão (2017) model with sunk entry costs. I consider two approaches to modeling ambiguity about the stochastic process for tariffs: multiple priors with max-min preferences (Gilboa and Schmeidler, 1989), in which firms act as if they face a transition matrix for tariffs that is "tilted" towards the worst-case outcome; and robust control (Hansen and Sargent, 2008), where firms treat the transition matrix as an approximation and form decision rules that perform well under perturbations of that matrix. I compare these approaches to the standard trade-policy risk setup, where firms have complete knowledge about the tariff process. Both forms of ambiguity lower export participation, acting like an increase in the likelihood and/or severity of a tariff increase under standard policy risk. They also change the comparative statics with respect to these parameters, making trade more sensitive to the latter and less sensitive to the former.

Second, I use a Alessandria et al. (2021) quantitative dynamic trade model to study the implications of policy ambiguity for aggregate trade dynamics. I simulate the model with different levels of tariff persistence and ambiguity and estimate trade-elasticity dynamics on each sample using the local projections method of Boehm et al. (2023). Without ambiguity, trade responds more in the long run to persistent tariff changes than transitory ones as shown by Alessandria et al. (2025b). When ambiguity is severe, trade responds similarly regardless of persistence. When it is mild, trade responds less to persistent tariff changes but actually responds more to transitory ones.

This paper contributes to the literature on trade-policy uncertainty, which studies how expectations about future trade policy affect trade flows in the present (Handley and Limão, 2017, 2022; Alessandria et al., 2025a,c). I add to this literature by comparing trade-policy risk, where tariffs follow a known stochastic process, with trade-policy ambiguity, where firms are uncertain in the Knightian sense about the nature of that process (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2008). I also contribute to the literature on trade-elasticity measurement, which reports a wide range of estimates across time horizons and contexts (?Simonovska and Waugh, 2014; Boehm et al., 2023; Alessandria et al., 2025b). I add to this literature by showing that ambiguity makes trade respond less to highly persistent trade reforms, but can also make it respond more to less persistent reforms. More broadly, this paper contributes to the literature on ambiguity and real-option decision problems. Outside trade, several papers have studied how ambiguity affects irreversible-investment and optimal-stopping problems (Nishimura and Ozaki, 2007; Thijssen, 2011; Huang and Yu, 2021). This is the first paper to study how ambiguity affects export participation.

# 2 Theory

I begin by analytically characterizing the effects of trade-policy ambiguity on export participation using the stylized economic model of Handley and Limão (2017). I consider three expectational setups: standard trade-policy risk where the distribution of future tariffs is known with certainty; Gilboa and Schmeidler (1989) max-min expected utility with multiple priors; and Hansen and Sargent (2008) robust control.

## 2.1 Environment

There is a unit measure of firms that are heterogeneous in productivity $z \sim F(z)$, which is exogenous, and export participation, which is endogenous. Firms begin their lives as non-exporters, die with probability $1-\delta$, and discount the future with a factor $\beta$. Exporting firms earn per-period profits

$$\pi_s(z) = Dz\tau_s^{1-\sigma} \equiv zT_s. \tag{1}$$

where $D$ is a constant aggregate demand term, and $\tau_s \geq 1$ is an ad-valorem tariff that depends on the aggregate state $s$, which I describe below, and $\sigma$ is the price elasticity of foreign demand for a firm's product. For notational brevity, I define $T_s = D\tau_s^{1-\sigma}$ as the profits of a firm with productivity one. Firms die with probability $1-\delta$ and are replaced by non-exporters, who can pay a one-time sunk cost $\kappa$ to become exporters.

For simplicity, I assume that tariffs follow a Markov process with two states, $s \in \{L, H\}$ with $\tau_L < \tau_H$, and transition matrix

$$P = \begin{bmatrix} 1-\eta & \eta \\ 1-\rho & \rho \end{bmatrix}, \tag{2}$$

where $\eta = \Pr(s' = H|s = L)$ is the hazard into high tariffs and $\rho = \Pr(s' = H|s = H)$ is the persistence of high tariffs. For future reference, the unconditional probability of the high-tariff state is

$$p_H = \frac{\eta}{\eta + 1 - \rho}. \tag{3}$$

## 2.2 Firm's problem

The problem of a non-exporter is to decide whether or not to begin exporting in order to maximize the expected present value of profits—or the certainty equivalent in the presence of ambiguity—net of the sunk cost. This problem is described by the following Bellman equations:

$$V_s^X(z) = \pi_s(z) + \delta\beta\mathcal{A}_s\left[V_{s'}^X(z)\right] \tag{4}$$

$$V_s^N(z) = \max\left\{\pi_s(z) - \kappa + \delta\beta\mathcal{A}_s\left[V_{s'}^X(z)\right],\ \delta\beta\mathcal{A}_s\left[V_{s'}^N(z)\right]\right\}, \tag{5}$$

where $V^X$ is the value of starting the period as an exporter, $V^N$ is the value of starting as a non-exporter, and $\mathcal{A}$ is the one-step certainty-equivalence operator specific to a given expectational setup. As in Handley and Limão (2017), the solution to this problem is characterized by a state-dependent threshold productivity $z_s^*$ where the value of entering equals the value of waiting:

$$\pi_s(z_s^*) - \kappa + \delta\beta\mathcal{A}_s\left[V_{s'}^X(z_s^*)\right] = \beta\delta\mathcal{A}_s\left[V_{s'}^N(z_s^*)\right] \tag{6}$$

The goal of this section is to characterize how the presence and extent of ambiguity affects these entry thresholds.

Regardless of the expectational setup, the entry threshold is higher in the high-tariff state than the low-tariff state: $z_H^* > z_L^*$. Thus, the firm productivity distribution is partitioned into three segments: firms below $z_L^*$ that never enter; firms between $z_L^*$ and $z_H^*$ that enter when tariffs are low and wait when tariffs are high; and firms above $z_H^*$ that always enter. For firms in the last category, the value of beginning the period as a non-exporter is equal to the value of starting as an exporter less the sunk entry cost in both states:

$$V_s^N(z_H^*) = V_s^X(z_H^*) - \kappa,\ s \in \{L, H\},\ z \geq z_H^*. \tag{7}$$

This implies that the equation that characterizes the high-tariff entry threshold can be written as

$$\pi_H(z_H^*) - \kappa + \delta\beta\mathcal{A}_H\left[V_{s'}^X(z_H^*)\right] = \beta\delta\mathcal{A}_H\left[V_{s'}^X(z_H^*) - \kappa\right]. \tag{8}$$

In all three setups, the one-step certainty-equivalence operator $\mathcal{A}$ satisfies translational invariance, i.e., adding a scalar to the value function state-by-state before evaluating the operator is equivalent to adding a scalar afterward: $\mathcal{A}\left[V + c\mathbf{1}\right] = \mathcal{A}\left[V\right] + c$. This implies that the continuation value drops out, and that the high-tariff threshold is equal to the ratio of the perpetually-amortized sunk cost to the high-tariff profit term in all three expectational setups:[1]

$$z_H^* = \frac{\kappa(1-\delta\beta)}{T_H}. \tag{9}$$

The condition that characterizes the low-tariff entry threshold can also be simplified by using the fact that the value of starting as a non-exporter is zero below this threshold: $V_s^N(z) = 0$ for $z \leq z_L^*$. This implies that at $z_L^*$, the sunk cost is exactly equal to the value of starting as an exporter:

$$\kappa = V_L^X(z_L^*) = \pi_L(z_L^*) + \delta\beta\mathcal{A}_L\left[V_{s'}^X(z_L^*)\right] \tag{10}$$

The remainder of this section focuses on characterizing how policy ambiguity affects this condition.

## 2.3 Solution with standard policy risk

The standard assumption in the trade-policy uncertainty literature is that while firms may not have perfect foresight over tariffs, they do know the true transition matrix $P$.[2] I refer to this setup as trade-policy risk. In this case, the certainty-equivalence operator is simply the expected value:

$$\mathcal{A}_s^{SR}\left[V_{s'}^X\right] = \mathbb{E}_P\left[V_{s'}^X\right] = \sum_{s'=L,H} P_{s,s'}V_{s'}^X. \tag{11}$$

Because firms never choose to exit after entering, we can apply this operator once to the entire infinite sequence of export profits, rather than recursively one period at a time.

To solve for $z_L^*$, we need to know the value of $V_L^X(z_L^*) = \pi_L(z_L^*) + \delta\beta\mathcal{A}_L\left[V_{s'}^X(z_L^*)\right]$. Matrix form is the easiest way to write this:

$$\begin{bmatrix} V_L^*(z_L^*) \\ V_H^*(z_L^*) \end{bmatrix} = \begin{bmatrix} \pi_L(z_L^*) \\ \pi_H(z_L^*) \end{bmatrix} + \mathcal{A}_L\begin{bmatrix} V_L^X(z_L^*) \\ V_H^X(z_L^*) \end{bmatrix} = (I-\delta\beta P)^{-1}\begin{bmatrix} z_L^* T_L \\ z_L^* T_H \end{bmatrix}. \tag{12}$$

The value for the low-state can be expressed in more economically intuitive fashion as

$$V_L^X(z_L^*) = z_L^*\left[\frac{\bar{T}}{1-\beta\delta} + \frac{T_L - \bar{T}}{1-\beta\delta(\rho-\eta)}\right], \tag{13}$$

[1] This result breaks if one departs from the Handley and Limão (2017) setup by adding a fixed cost of continuing to export and making firm productivity stochastic, which makes closed-form characterization impossible. Adding these quantitative bells and whistles and investigating numerically how they interact with trade-policy ambiguity is the purpose of the next section.

[2] Studies that allow the policy transition matrix to vary over time, such as Alessandria et al. (2025a,c), either assume that firms know the entire path of transition matrices with certainty, or that they know the current matrix at any given point in time and believe it will last forever.

Thus, (10) simplifies to

$$z_L^{*,SR} = \frac{\kappa}{\left[\frac{\bar{T}}{1-\beta\delta} + \frac{T_L-\bar{T}}{1-\beta\delta(\rho-\eta)}\right]}, \tag{14}$$

The properties of the threshold $z_L^*$ under standard trade-policy risk are intuitive and well-known. It is increasing in the sunk entry cost, $\kappa$, which makes entry less attractive, and it is increasing in the high-tariff value, $\tau_H$, the probability of switching from the low-tariff state to the high-tariff state, $\eta$, and the probability of staying in the high-tariff state, $\rho$, all of which increase the expected present value of tariffs when starting in the low-tariff state.

## 2.4 Solution with max-min policy ambiguity

The Gilboa and Schmeidler (1989) approach to modeling ambiguity assumes that firms do not know the transition matrix $P$, but instead have a set of possible priors about that matrix and act as if they faced the worst possible element in that set. The one-step operator in this setup is

$$\mathcal{A}_s^{MM}\left[V_{s'}^X;\alpha\right] = \inf_{P'\in\mathcal{P}(\alpha)} \mathbb{E}_{P'}\left[V_{s'}^X\right] = \inf_{P'\in\mathcal{P}} \sum_{s'=L,H} P_{s,s'}V_{s'}^X. \tag{15}$$

The degree of ambiguity $\alpha$ controls the extent to which the set of priors $\mathcal{P}(\alpha)$ is allowed to diverge from the true matrix. In Markov settings the typical assumption is that this divergence is "rectangular:"

$$\mathcal{P}(\alpha) = \{(1-\alpha)P(s,\cdot) + \alpha\tilde{s} : \tilde{s} \in \{L,H\}\},\ \alpha \in [0,1]. \tag{16}$$

This implies that the firm simply puts the additional weight $\alpha$ on the worst-case outcome $\tau_H$ in both states, acting as if it faces the "tilted" transition matrix,

$$P'(\alpha) = \begin{bmatrix} 1-\eta'(\alpha) & \eta'(\alpha) \\ 1-\rho'(\alpha) & \rho'(\alpha) \end{bmatrix}, \tag{17}$$

where $\eta'(\alpha) = \eta + \alpha(1-\eta)$ and $\rho'(\alpha) = \rho + \alpha(1-\rho)$.

The solution in this setup takes the same form as (14), with the elements of $P$ replaced by the analogous elements of $P'(\alpha)$. The value of starting as an exporter in the low-tariff state is now

$$V_L^X(z_L^*(\alpha);\alpha) = \frac{\bar{T}(\alpha)}{1-\beta\delta} + \frac{T_s - \bar{T}(\alpha)}{1-\beta\delta\left(\rho'(\alpha)-\eta'(\alpha)\right)}, \tag{18}$$

where $\bar{T}(\alpha) = p'_H(\alpha)T_H + (1-p'_H(\alpha))T_L$ is the unconditional average of $T_s$ under the worst-case transition matrix $P'(\alpha)$. The entry threshold is

$$z_L^{*,MM} = \frac{\kappa}{\frac{\bar{T}(\alpha)}{1-\beta\delta} + \frac{T_s-\bar{T}(\alpha)}{1-\beta\delta(\rho'(\alpha)-\eta'(\alpha))}}. \tag{19}$$

The threshold takes the same form as under standard policy risk but is increasing in the degree of ambiguity, $\alpha$. When $\alpha$ increases, the expected present value of tariffs under the worst-case prior $P'(\alpha)$ increases, because the probability of switching to the high-tariff state, $\eta'(\alpha)$, and the probability of staying in that state, $\rho'(\alpha)$, both rise. When $\alpha = 0$, this setup coincides with standard risk. When $\alpha = 1$, the firm acts as if it will face the high-tariff state in every period after the current one. Formally,

$$z_L^{*,MM} > z_L^{*,SR} \; \forall \alpha > 0, \quad \frac{\partial z_L^{*,MM}}{\partial \alpha} > 0, \; \forall \alpha \in [0, 1). \tag{20}$$

### 2.5 Solution with robust control

Hansen and Sargent (2008) propose an ambiguity model that uses robust control theory to discipline the set of candidate transition matrices $\mathcal{P}$. The basic idea is that the firm treats $P$ as an approximation and forms a decision rule that performs well under perturbations of $P$. They formalize this idea by positing a two-player problem in which "nature" seeks to minimize the firm's payoff by perturbing the transition matrix but pays a penalty proportional to the relative entropy between its choice and the true matrix.[3] In this setup, the one-step operator is

$$\mathcal{A}_s^{RC}\left[V_{s'}^X; \theta\right] = -\theta \log \left[\sum_{s'=L,H} P_{s,s'} e^{-V_{s'}^X/\theta}\right]. \tag{21}$$

The parameter $\theta$ is the entropy penalty. When $\theta$ goes to zero, this setup is equivalent to max-min ambiguity with $\alpha = 1$. When $\theta$ is sufficiently large, this operator can be approximated by a second-order expansion:

$$\mathcal{A}_s^{RC}\left[V_{s'}^X; \theta\right] \approx \mathbb{E}_P\left[T_{s'}|s\right] - \frac{1}{2\theta} \operatorname{Var}_P\left[V_{s'}^X|s\right]. \tag{22}$$

As in the other two setups, we can still apply this operator once to the stream of export profits after entering.

The value of starting as an exporter in the low-tariff state can be approximated as a quadratic function $z_L^*$,

$$V^* X_L(z_L^*) \approx b_L z_L^* - \frac{a_L}{2\theta}(z_L^*)^2, \tag{23}$$

[3] This setup is closely related to the "smooth ambiguity" approach of Klibanoff et al. (2005), who propose a decision maker with a set of possible priors about $P$, a "second-order" distribution $\mu$ that assigns a probability to each of these priors, and ambiguity aversion captured by a general nonlinear function $\phi$ that aggregates outcomes across these priors. When $\phi$ takes the exponential form in (21), the two setups are equivalent.

where

$$\begin{bmatrix} b_L \\ b_H \end{bmatrix} := (I - \delta\beta P)^{-1} \begin{bmatrix} T_L \\ T_H \end{bmatrix}, \quad \begin{bmatrix} a_L \\ a_H \end{bmatrix} := \delta\beta(I - \delta\beta P)^{-1} \begin{bmatrix} \eta(1-\eta)(b_H - b_L)^2 \\ \rho(1-\rho)(b_H - b_L)^2 \end{bmatrix}. \tag{24}$$

The low-state threshold is the solution to

$$\frac{a_L}{2\theta}(z_L^*)^2 - b_L z^* L + \kappa = 0, \tag{25}$$

which is given by

$$z_L^{*,RC}(\theta) = \frac{b_L - \sqrt{b_L^2 - 2a_L\kappa/\theta}}{a_L/\theta}. \tag{26}$$

The first-order approximation of this solution is more economically transparent:

$$z_L^{*,RC} \approx \frac{\kappa}{b_L} + \frac{a_L\kappa^2}{2\theta b_L^3}. \tag{27}$$

It shows that the robust-control solution coincides with the solution under standard risk when there is no ambiguity (i.e., the entropy penalty $\theta$ is infinite), and increases with the degree of ambiguity (i.e., increases when $\theta$ falls), similar to the max-min setup. However, there is an additional nonlinear term which is increasing in the degree of ambiguity. Formally,

$$z_L^{*,RC} > z_L^{*,SR}\ \forall\theta \in \mathbb{R}_+, \quad \frac{\partial z_L^{*,RC}}{\partial\theta} < 0,\ \forall\theta \in \mathbb{R}_+. \tag{28}$$

## 2.6 Comparative statics

Thus far, we have shown that trade-policy ambiguity depresses export participation. A more interesting question is how trade-policy ambiguity interacts with the model's other parameters. For example, is the sunk cost a greater or lesser deterrent to exporting under trade-policy ambiguity? Does ambiguity amplify or attenuate the effect of an increase in the probability of switching to the high-tariff state? All proofs of the results discussed below are relegated to the appendix.

### 2.6.1 Sunk entry cost ($\kappa$)

The sunk entry cost is the key technological parameter that governs export participation. In all three setups, the higher the sunk cost, the more productive a firm must be to enter:

$$\frac{\partial z_L^{*,SR}}{\partial\kappa} > 0, \quad \frac{\partial z_L^{*,MM}}{\partial\kappa} > 0 \forall\alpha \in [0,1], \quad \frac{\partial z_L^{*,RC}}{\partial\kappa} > 0\ \forall\theta \in \mathbb{R}_+. \tag{29}$$

Because policy ambiguity reduces the certainty-equivalent of export profits, it makes the entry threshold more sensitive to the sunk cost:

$$\frac{\partial z_L^{*,MM}}{\partial \kappa} > \frac{\partial z_L^{*,SR}}{\partial \kappa} \; \forall \alpha \in (0,1], \qquad \frac{\partial z_L^{*,RC}}{\partial \kappa} > \frac{\partial z_L^{*,SR}}{\partial \kappa} \; \forall \theta \in \mathbb{R}_+. \tag{30}$$

A corollary is that a marginal increase in the sunk cost has a larger impact on exporting when ambiguity is stronger:

$$\frac{\partial^2 z_L^{*,MM}}{\partial \alpha \partial \kappa} > 0 \; \forall \alpha \in (0,1], \qquad \frac{\partial^2 z_L^{*,RC}}{\partial \theta \partial \kappa} < 0 \; \forall \theta > 0. \tag{31}$$

Under standard risk and max-min ambiguity, the threshold increases linearly with the sunk cost. However, under robust control, there is an additional quadratic term:

$$\frac{\partial^2 z_L^{*,SR}}{\partial \kappa^2} = 0, \quad \frac{\partial^2 z_L^{*,MM}}{\partial \kappa^2} = 0, \quad \frac{\partial^2 z_L^{*,RC}}{\partial \kappa^2} < 0 \; \forall \theta > 0. \tag{32}$$

This is because robust control penalizes the variance in future payoffs, whereas the firm's objective in the max-min setup is still linear in the payoff stream, just as in the standard risk setup. As the marginal entrant gets more productive—which is what happens when $\kappa$ rises—the gap between the marginal entrant's payoffs under low and high tariffs rises, and this amplifies the cost of ambiguity under robust control.

These comparative statics show that trade-policy ambiguity has heterogeneous effects across industries, depressing trade more in industries with greater sunk entry costs, particularly in the robust-control setup. This is qualitatively similar to the way trade-policy risk bites more when the sunk cost is larger in Handley and Limão (2017); increases in the probability of bad trade-policy outcomes have similar effects to increases in the ambiguity about the distribution of future outcomes. I explore the interaction between trade-policy risk and trade-policy ambiguity below.

### 2.6.2 High tariff value ($\tau_H$)

The high-tariff value also plays a key role in driving export participation. In all three setups, the more tariffs can rise, the more productive a firm must be to make entry profitable:

$$\frac{\partial z_L^{*,SR}}{\partial \tau_H} > 0, \quad \frac{\partial z_L^{*,MM}}{\partial \tau_H} > 0 \; \forall \alpha \in [0,1], \quad \frac{\partial z_L^{*,RC}}{\partial \tau_H} > 0 \; \forall \theta \in \mathbb{R}_+. \tag{33}$$

Trade-policy ambiguity makes the potential increase in tariffs more important:

$$\frac{\partial z_L^{*,MM}}{\partial \tau_H} > \frac{\partial z_L^{*,SR}}{\partial \tau_H} \; \forall \alpha \in (0,1], \qquad \frac{\partial z_L^{*,RC}}{\partial \tau_H} > \frac{\partial z_L^{*,SR}}{\partial \tau_H} \; \forall \theta \in \mathbb{R}_+. \tag{34}$$

In the max-min setup, this is because there is more weight on the high-tariff state in the worst-case transition matrix $P'(\alpha)$. In the robust-control setup, a greater potential tariff increase reduces the mean profits from exporting, $\mathbb{E}_P\left[T_{s'}|s\right]$, and also increases the variance penalty, $\frac{1}{2\theta}\operatorname{Var}_P\left[V^X_{s'}|s\right]$. As with the sunk cost, changes in the value of the high tariff have more impact on exporting when the degree of ambiguity is greater:

$$\frac{\partial^2 z_L^{*,MM}}{\partial\alpha\partial\tau_H} > 0\ \forall\alpha\in(0,1], \qquad \frac{\partial^2 z_L^{*,RC}}{\partial\theta\partial\tau_H} < 0\ \forall\theta > 0. \tag{35}$$

These comparative statics show that tariff bindings and legislative caps on tariffs have a greater impact on trade when trade policy is ambiguous. When the likelihood of a tariff increase is known with certainty (i.e., the standard trade-policy risk setup), increases in the maximum tariff value depress trade less than when this likelihood is unknown (i.e., the degree of ambiguity is higher). Conversely, lowering the maximum tariff—for example, in the case of the United States, passing legislation to limit tariff powers delegated to the President under the Trade Act and the Trade Expansion Act—boosts trade more when the degree of ambiguity is higher.

### 2.6.3 Likelihood and persistence of high tariffs $(\eta,\rho)$

In all three setups, the more likely tariffs are to rise and the more persistent such an increase is, the more productive a firm must be to enter when tariffs are low:

$$\frac{\partial z_L^{*,SR}}{\partial p} > 0, \quad \frac{\partial z_L^{*,MM}}{\partial p} > 0\ \forall\alpha\in[0,1), \quad \frac{\partial z_L^{*,RC}}{\partial p} > 0\ \forall\theta\in\mathbb{R}_+,\ p=\eta,\rho. \tag{36}$$

Thus, increases in trade-policy risk always reduce trade, regardless of whether that risk is ambiguous or not (except for in the max-min setup when $\alpha = 1$, as I discuss below). A more interesting question is how policy ambiguity modulates the effects of policy risk. Unlike the comparative statics with respect to the sunk cost and the tariff bound, where ambiguity always makes trade more sensitive, the results here are more nuanced. In some circumstances, ambiguity amplifies the effects of changes in $\eta$ and $\rho$ on trade, but in others it attenuates them.

In the max-min setup, there exists a threshold value $\bar{\alpha}_p$ for each transition probability $p=\eta,\rho$ such that $z_L^{*,MM}$ is less sensitive than $z_L^{*,SR}$ to changes in that probability for $\alpha > \bar{\alpha}_p$. The intuition is that as $\alpha$ goes to one, the firm acts as if it will always face the high-tariff state in the future regardless of $\eta$ and $\rho$ and does not react to changes in these parameters. Below the threshold $\bar{\alpha}_p$, $z_L^*$ is more sensitive to changes in $p$ than under standard risk. Formally,

$$\lim_{\alpha\to1}\frac{\partial z_L^{*,MM}}{\partial p} = 0, \quad \exists\bar{\alpha}_p\in[0,1)\ \operatorname{sign}\left(\frac{\partial z_L^{*,MM}}{\partial p} - \frac{\partial z_L^{*,SR}}{\partial p}\right) = \operatorname{sign}\left(\bar{\alpha}_p-\alpha\right), \quad p=\eta,\rho \tag{37}$$

For $p = \eta$, $\bar{\alpha}_\eta = 0$ in all parameterizations: $z_L^{*,MM}$ is always less sensitive than $z_L^{*,SR}$ to changes in $\eta$ under max-min ambiguity, and $\partial z_L^{*,MM}/\partial p$ converges monotonically downward to zero as $\alpha$ goes to one. For $p = \rho$, on the other hand, there are some parameterizations where $z_L^*$ is always less sensitive to $\rho$ than under standard risk (i.e., $\bar{\alpha}_\rho = 0$), and there are others where it is more sensitive to $\rho$ for low levels of ambiguity and less sensitive for higher levels (i.e., $\bar{\alpha}_\rho > 0$).

Figure 1 illustrates this using a numerical example. Panel (a) plots the ratio of $\partial z_L^{*,MM}/\partial\eta$ to $\partial z_L^{*,SR}/\partial\eta$ as a function of $\alpha$ for several different values of $\eta$. In all cases, this ratio equals one for $\alpha = 0$ and declines as $\alpha$ rises. Panel (b) plots the ratio of $\partial z_L^{*,MM}/\partial\rho$ to $\partial z_L^{*,SR}/\partial\rho$. When $\eta$ is low (i.e., high tariffs are unlikely), the entry threshold is more sensitive to $\rho$ under max-min ambiguity than standard risk unless $\alpha$ is very high. When $\eta$ is sufficiently high, the entry threshold is less sensitive to $\rho$ under max-min ambiguity for all values of $\alpha$. In general, varying $\rho$ while holding fixed $\eta$ changes the location of $\bar{\alpha}_\rho$, but does not change whether $\bar{\alpha}_\rho > 0$.

Under robust control, the entry threshold can also be more or less sensitive to the transition probabilities than under standard risk, but the degree of ambiguity does not play a role; only the transition probabilities themselves do. One can show that

$$\operatorname{sign}\left(\frac{\partial z_L^{*,RC}}{\partial\eta} - \frac{\partial z_L^{*,SR}}{\partial\eta}\right) = \operatorname{sign}\left(\frac{1}{\eta} - \frac{3\gamma}{1-\gamma(\rho-\eta)} - \frac{1-\gamma\rho}{(1-\gamma\rho(1-\eta)+\gamma\rho(1-\rho)}\right) \tag{38}$$

$$\operatorname{sign}\left(\frac{\partial z_L^{*,RC}}{\partial\rho} - \frac{\partial z_L^{*,SR}}{\partial\rho}\right) = \operatorname{sign}\left(\frac{3}{1-\gamma(\rho-\eta)} - \frac{2\rho-\eta}{(1-\gamma\rho(1-\eta)+\gamma\rho(1-\rho)}\right). \tag{39}$$

One can further show that if the right-hand side of (38) is positive, so is the right-hand side of (39). Thus, the $(\eta, \rho)$ plane is divided into three regions: one where $z_L^*$ is more sensitive to both parameters under robust control than under standard risk; another where $z_L^*$ is more sensitive only to $\rho$; and a third where $z_L^*$ is less sensitive to both parameters. Figure 2 provides an illustration.

To sum up, trade-policy ambiguity can amplify or attenuate the effects of trade-policy risk. In the max-min setup, what matters is the degree of ambiguity; trade is more sensitive to increases in the persistence of high tariffs at low levels of ambiguity and less sensitive for higher levels. Under robust control, only the baseline hazard and persistence of high tariffs matter; when a tariff hike is unlikely ($\eta$ is low) but persistent ($\rho$ is high), ambiguity makes trade more sensitive to changes in trade-policy risk.

#### 2.6.4 Idiosyncratic productivity ($z$)

Max-min and robust control ambiguity have similar effects on export participation and the manner in which it depends on the model's parameters. However, there is one way in which the two approaches to modeling trade-policy ambiguity work differently: max-min ambiguity affects all firms in the same way, but high-productivity firms are more sensitive to ambiguity under robust-control.

To see this, first note that the one-step operator $\mathcal{A}_s$ under robust control can be written as

$$\mathcal{A}_s^{RC}\left[V_{s'}^X;\theta\right] = \sum_{s'=L,H} \tilde{P}_{s,s'}(z;\theta)V_{s'}^X + \theta \sum_{s'=L,H} \tilde{P}_{s,s'}(z;\theta)\log\left[\frac{\tilde{P}_{s,s'(z;\theta)}}{P_{s,s'}}\right] \tag{40}$$

with "tilted" probabilities given by

$$\tilde{P}_{s,s'}(z;\theta) = \frac{P_{s,s'}e^{-V_{s'}^X(z)/\theta}}{\sum_{r=L,H} P_{s,r}e^{-V_r^X(z)/\theta}} \tag{41}$$

that depend on productivity $z$ as well as the degree of ambiguity $\theta$. We can write the tilted odds ratio of the high-tariff state as

$$\frac{\tilde{P}_{s,H}(z;\theta)}{\tilde{P}_{s,L}(z;\theta)} = \frac{P_{s,H}}{P_{s,L}}e^{\left[V_L^X(z)-V_H^X(z)\right]/\theta} := \frac{P_{s,H}}{P_{s,L}}e^{\Delta V^X(z)/\theta} \tag{42}$$

The value gap $\Delta V^X(z)$ is increasing in $z$, and so the odds ratio is as well:

$$\frac{\partial}{\partial z}\log\left[\frac{\tilde{P}_{s,H}(z;\theta)}{\tilde{P}_{s,L}(z;\theta)}\right] > 0. \tag{43}$$

Thus, high-productivity firms put relatively more weight on the high-tariff state than low-productivity firms under robust control. By contrast, under max-min ambiguity, all firms put the same additional weight $\alpha$ on the high-tariff state.

In the simple model I have analyzed in this section the entry decision is the only margin of interest so there is no scope for this difference to affect any other outcomes, such as exit, firm growth, etc. In a quantitative model with ingredients like endogenous exit and exporter life cycles, this difference becomes more important. I take that up in the next section of the paper.

# 3 Quantification

I now use a quantitative dynamic trade model to explore the implications of trade-policy ambiguity for trade adjustment dynamics.

## 3.1 Model

The model is a partial-equilibrium version of Alessandria et al. (2021). The key differences relative to the simple model used in the previous section are fixed costs to continue exporting and idiosyncratic shocks to productivity and variable trade costs. Together, these additional ingredients generate endogenous exit and gradual trade growth at both the micro and macro levels.

### 3.1.1 Production and demand

Firms use labor to produce output according to constant returns to scale technology:

$$y_t = z_t \ell_t. \tag{44}$$

Productivity evolves over time according to an AR(1) process in logs:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t, \tag{45}$$

where $\varepsilon_t$ is i.i.d. across firms and time. Firms produce differentiated goods and compete monopolistically. Foreign demand for a firm's good is a downward-sloping function of the price, $p_t$, and the import tariff, $\tau_t$:

$$d_t(p_t, \tau_t) = (p_t \tau_t)^{-\sigma}, \tag{46}$$

where $\sigma$ is the price elasticity of demand.

### 3.1.2 Trade costs and life cycles

There are three types of trade costs: import tariffs, variable trade costs, and fixed trade costs. The import tariff is an aggregate state variable that follows the same two-state Markov process described in 2.1.

Variable trade costs are firm-specific and can take three values, $\infty > \xi_H > \xi_L$. The fixed trade cost is a function of the iceberg cost. To begin exporting, a non-exporter with $\xi = \infty$ must pay a fixed cost $\kappa(\infty) = \kappa_0$. To continue exporting, a low- or high-cost exporter must pay a fixed cost $\kappa(\xi_H) = \kappa(\xi_L) = \kappa_1$.

Firms are born as non-exporters. When a firm begins exporting, its variable trade cost falls to $\xi_H$ in the next period. When a high-cost exporter chooses to continue exporting, its variable cost has a chance $1-\rho_\xi$ of falling to $\xi_L$ in the next period. Symmetrically, a low-cost exporter retains its value of $\xi$ with probability $\rho_\xi$. In each period, firms die with probability $1-\delta(z) = \max\left[0, \min\left(e^{-\delta_0 z} + \delta_1, 1\right)\right]$, in which case they are replaced by non-exporters with productivities drawn from the ergodic distribution.

### 3.1.3 Firm's problem

The firm's state variables are its productivity, $z$, its variable trade cost, $\xi$, and the tariff, $\tau$. The firm's problem has a static component and a dynamic component. The static problem entails choosing a price to maximize the flow profits from exporting given the current state:

$$\pi(z,\xi,\tau) = \max_{p} \left\{ pd(p,\tau) - w\frac{\xi d(p,\tau)}{z_t} \right\}. \tag{47}$$

The dynamic problem entails deciding whether to export in the next period. The value of a firm that chooses to export in $t+1$ is

$$V^X(z,\xi,\tau) = -\kappa(\xi) + \frac{\delta(z)}{1+r}\mathcal{A}_\tau \left\{ \mathbb{E}_{z',\xi'} \left[ V(z',\xi',\tau') \right] \right\}, \tag{48}$$

and the value of a firm that chooses not to export is

$$V^N(z,\xi,\tau) = \frac{\delta(z)}{1+r}\mathcal{A}_\tau \left\{ \mathbb{E}_{z'} \left[ V(z',\infty,\tau') \right] \right\}, \tag{49}$$

where

$$V(z,\xi,\tau) = \pi(z,\xi,\tau) + \max\{V^N(z,\xi,\tau), V^X(z,\xi,\tau)\}. \tag{50}$$

Note that the firm takes expectations about its idiosyncratic state variables, $z$ and $\xi$, before applying the one-step operator $\mathcal{A}$; there is no ambiguity about idiosyncratic shocks, only the aggregate tariff state. Similar to the simple model in the previous section, the solution to the dynamic problem is characterized by a threshold productivity, $z^*(\xi,\tau)$, but it now depends on the firm's variable trade cost and the tariff state.

#### 3.1.4 Aggregation

Aggregate exports in time $t$ depend on the entire history of tariff realizations, $\{\tau_s\}_{s=0}^t$. The distribution of firms over the idiosyncratic state, $\varphi_t(z,\xi)$, depends on the tariff history up to the end of the previous period:

$$\varphi_t(\mathcal{Z},\infty) = \sum_{\xi} \left[ \int_0^{z^*(\xi,\tau_{t-1})} h(\mathcal{Z},z)\varphi_{t-1}(z,\xi)dz + \int_0^{\infty} \bar{h}(\mathcal{Z})\varphi_{t-1}(z,\xi)dz \right], \tag{51}$$

$$\begin{aligned} \varphi_t(\mathcal{Z},\xi_H) = & \int_{z^*(\infty,\tau_{t-1})}^{\infty} h(\mathcal{Z},z)\varphi_{t-1}(z,\infty)dz + \rho_\xi \int_{z^*(\xi_H,\tau_{t-1})}^{\infty} h(\mathcal{Z},z)\varphi_{t-1}(z,\xi_H)dz \\ & + (1-\rho_\xi) \int_{z^*(\xi_L,\tau_{t-1})}^{\infty} h(\mathcal{Z},z)\varphi_{t-1}(z,\xi_L)dz, \end{aligned} \tag{52}$$

$$\varphi_t(\mathcal{Z},\xi_L) = (1-\rho_\xi) \int_{z^*(\xi_H,\tau_{t-1})}^{\infty} h(\mathcal{Z},z)\varphi_{t-1}(z,\xi_H)dz + \rho_\xi \int_{z^*(\xi_L,\tau_{t-1})}^{\infty} h(\mathcal{Z},z)\varphi_{t-1}(z,\xi_L)dz, \tag{53}$$

where $\mathcal{Z}$ is a typical subset of $\mathbb{R}_{++}$, $h(\mathcal{Z},z)$ is the probability of surviving and drawing a new productivity in $\mathcal{Z}$ conditional on today's productivity $z$ under the process (45), and $\bar{h}(\mathcal{Z})$ is the probability of dying and being replaced by a new firm with productivity in $\mathcal{Z}$.

Aggregate exports depend on the distribution $\varphi_t$ and the current tariff:

$$X = \sum_{\xi \in \{\xi_L, \xi_H\}} \int_z p(z, \xi, \tau_t) d(p_t(z, \xi, \tau_t), \tau_t) \, \mathrm{d}\varphi_t(z, \xi). \tag{54}$$

### 3.1.5 Parameter values

I use the parameter values that Alessandria et al. (2025b) calibrated to match Vietnamese firm-level data. As that paper shows, these values imply very large long-run trade responses to persistent trade reforms, which we observe very rarely, but much smaller responses to transitory reforms, which we observe much more frequently. Table 1 below lists these values.

One parameter to take particular note of is $\sigma$, the demand elasticity, which is the elasticity of trade to tariffs in the short run. The value for this parameter was originally taken from **?**. There is a great deal of disagreement in the empirical literature about the trade elasticity, with recent estimates ranging from less than two (Boehm et al., 2023) to greater than ten (Alessandria et al., 2025b). As Alessandria et al. (2025b) show, one potential explanation for this disagreement is that trade responds less to transitory tariff shocks than permanent ones, and while quantitative analyses focus on the latter, trade data consist mostly of the former. In this paper, I build on this idea and show that ambiguity about the tariff process can also help explain the dispersion in empirical estimates.

## 3.2 Experiments

To study how trade-policy ambiguity affects aggregate trade, I estimate trade-elasticity dynamics using simulated data generated by the model. Following Alessandria et al. (2025a,c,b), I assume now that there is a large number of products indexed by $g = 1, \ldots, G$, each with its own continuum of firms operating as described in section 3.1 above, and that tariff shocks are independent across products but common across all firms producing the same product. For a given parameterization, I simulate the model for a large number of periods $t = 1, \ldots, T$, resulting in a panel dataset at the product-time level with $G \times T$ observations.

Using this simulated data, I estimate the trade elasticities using local projections following Boehm et al. (2023). The empirical specification is

$$\Delta_h X_{gt} = -\beta_h^X \Delta_h \tau_{gt} + \delta_{gt} + u_{gt}, \tag{55}$$

where $\Delta_h$ is an operator that takes the log difference between a variable's value in period $t + h$ and its value in period $t - 1$. The $h$-period tariff change $\Delta_h \tau_{gt}$ is instrumented using the one-period change:

$$\Delta_h \tau_{gt} = \beta_h^\tau \Delta_0 \tau_{gt} + \delta_{gt} + u_{jgt}. \tag{56}$$

I simulate the model and run the estimation procedure separately for different combinations of tariff persistence, $\rho$, and the degree of ambiguity, $\alpha$ or $\theta$, to study how $\beta_h^X$ varies as these parameters change. I set the maximum horizon to 14. Beyond this value, estimates lose stability for extremely persistent reforms because tariffs change very infrequently.

### 3.3 Results

Figure 3 shows the results. The graphs in the left column of the figure show the results for max-min ambiguity. When tariffs are less persistent ($\rho = 0.8$), the long-run response of trade is similar regardless of the degree of ambiguity; even under standard policy risk, firms do not respond much to transitory reforms because they know they are likely to be reversed as in (Alessandria et al., 2025b). When tariffs are more persistent ($\rho = 0.95$ and $\rho = 0.99$), ambiguity attenuates the long-run trade response to a greater degree, because firms act as if tariff reductions are more likely to be reversed than they really are. When ambiguity is severe ($\alpha = 0.75$), the long-run trade response is similar regardless of the true level of tariff persistence because firms always act as if a switch from low tariffs to high tariffs is likely to occur and likely to last.

The right column of the figure shows the results for robust control. When tariff persistence is extremely high ($\rho = 0.99$), the results are similar to max-min ambiguity. When $\rho = 0.95$, the results are similar if ambiguity is severe (i.e., $\theta$ is lower), but trade responds more in the medium run under mild levels of ambiguity (i.e., when $\theta$ is higher) than under standard risk, although it responds slightly less in the long run. When tariffs are transitory ($\rho = 0.8$), the trade response under mild levels of ambiguity is substantially larger than under standard risk at all horizons, but when ambiguity is severe enough the response is smaller.

Figure 4 summarizes these results by plotting the ratio of the long-run trade elasticity estimate, $\beta_{14}^X$, to the short-run estimate, $\beta_0^X$. For less persistent reforms ($\rho = 0.8$), mild trade-policy ambiguity amplifies the long-run trade response, especially under robust control, but strong ambiguity attenuates it. For more persistent reforms ($\rho = 0.95$ and $\rho = 0.99$), on the other hand, the long-run trade response is always smaller under trade-policy ambiguity than standard risk. In general, trade-policy ambiguity makes aggregate trade less sensitive to the true level of tariff persistence: the differences in trade responses between low- and high-persistence tariff processes are smaller under trade-policy ambiguity than under standard risk, for both mild and strong degrees of ambiguity. This is particularly true for the robust-control setup, where mild ambiguity induces a significantly larger response to low-persistence reforms and a significantly smaller response to high-persistence reforms.

# 4 Conclusion

This paper compares the effects of trade-policy risk, where future tariffs are uncertain but their distribution is known, with trade-policy ambiguity, where the distribution itself is uncertain. Theoretically, I show that policy risk and ambiguity both reduce trade, but the latter also makes trade more sensitive to entry costs and tariff bounds, and usually makes trade less sensitive to tariff persistence. Quantitatively, I show that these effects manifest in weaker responses of aggregate trade to persistent tariff changes but stronger responses to transitory changes.

My findings add to the conversation about the magnitude of the trade elasticity by illustrating how ambiguity can reduce measured trade responses, but also makes trade respond similarly to transitory and persistent reforms. They also underscore the importance of credible and enforceable rules that limit the size of tariff increases, like WTO bindings and legislative constraints on executive tariff powers. These rules and institutions are more important when ambiguity is salient, so strengthening them is more important than ever in the current geopolitical environment.

# 5 Declaration of generative AI use

During the preparation of this work the author used ChatGPT5 Pro to prove mathematical results more efficiently and debug computer code. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article. Links to the author's conversations with ChatGPT can be found here, here, and here.

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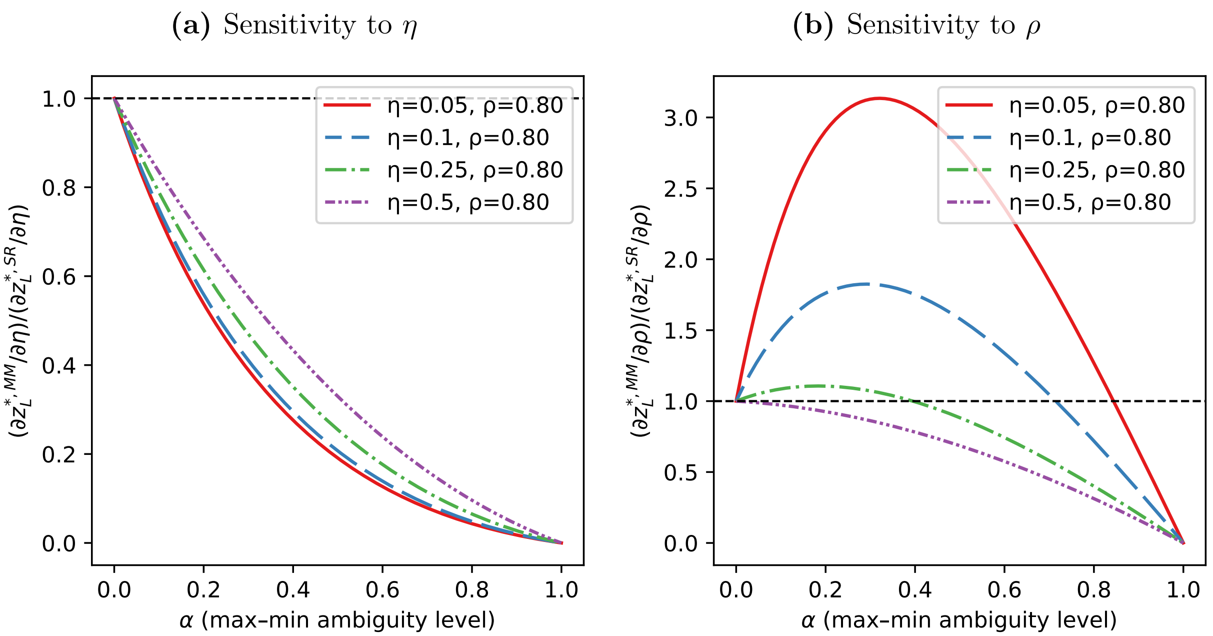
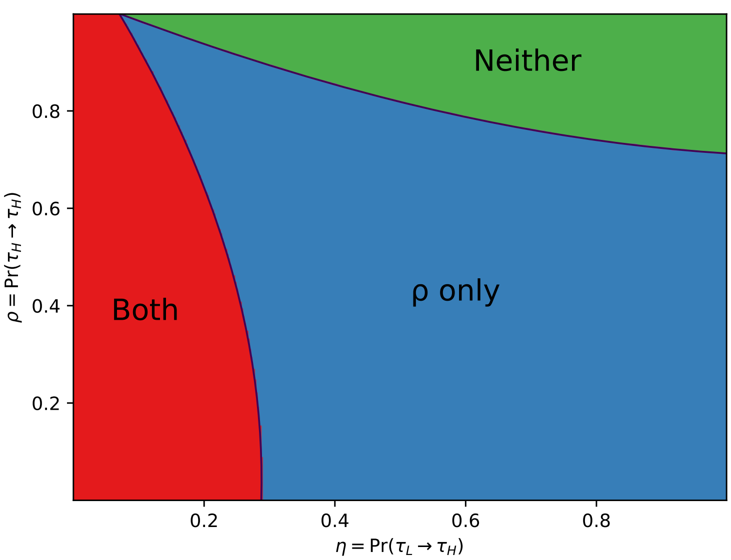
**Figure 1:** Sensitivity of $z_L^*$ under max-min ambiguity to transition probabilities



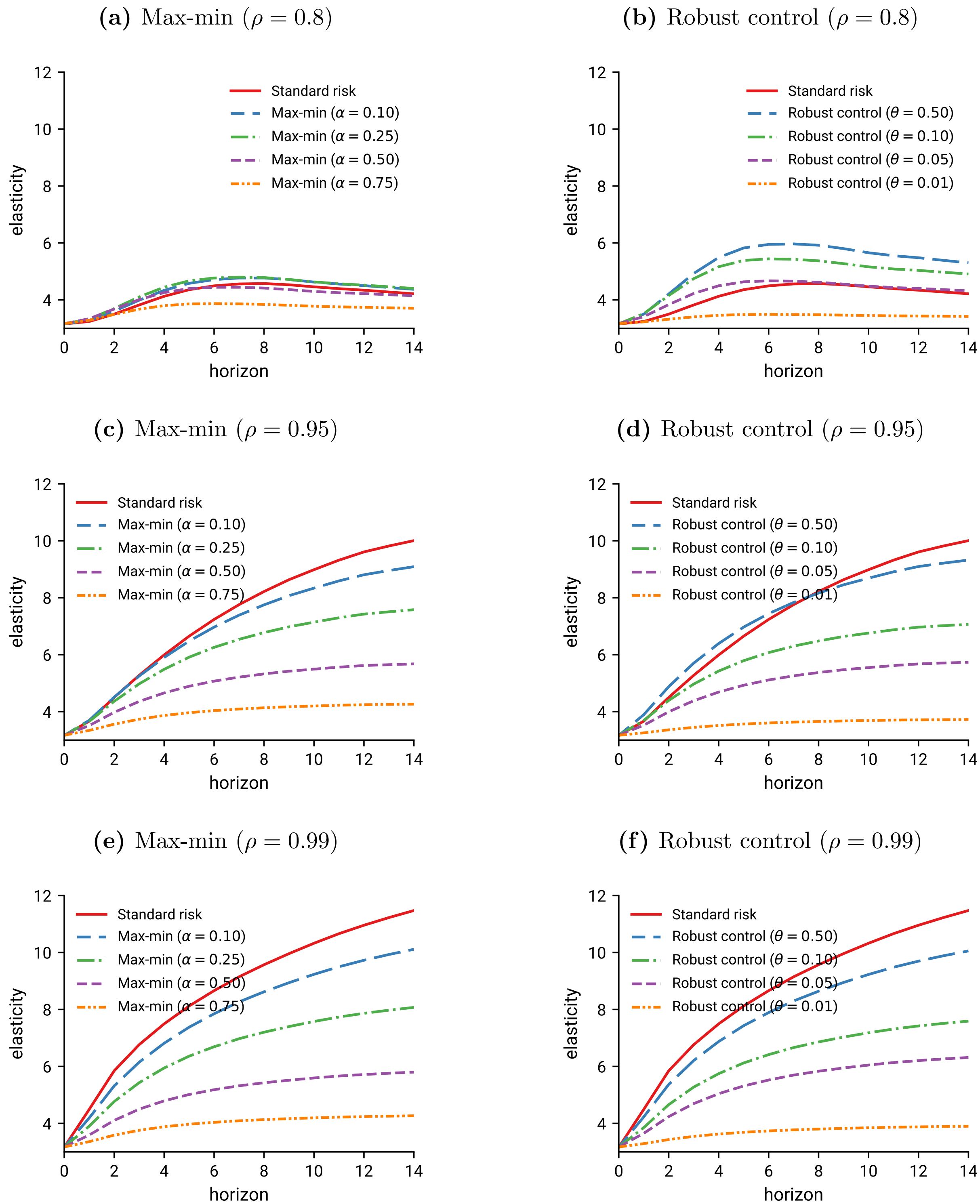

*Notes:* (a) Partial derivative of $z_L^*$ with respect to $\eta$ under max-min ambiguity relative to same partial derivative under standard risk, as a function of the degree of ambiguity $\alpha$. (b) Same as (a), but with respect to $\rho$. In both panels, $\beta = 0.96$, $\delta = 0.9$, $\sigma = 4$, $\tau_L = 1.0$, and $\tau_H = 1.25$.

**Figure 2:** Parameterizations where $z_L^*$ is more sensitive under robust control to $\eta$ and $\rho$



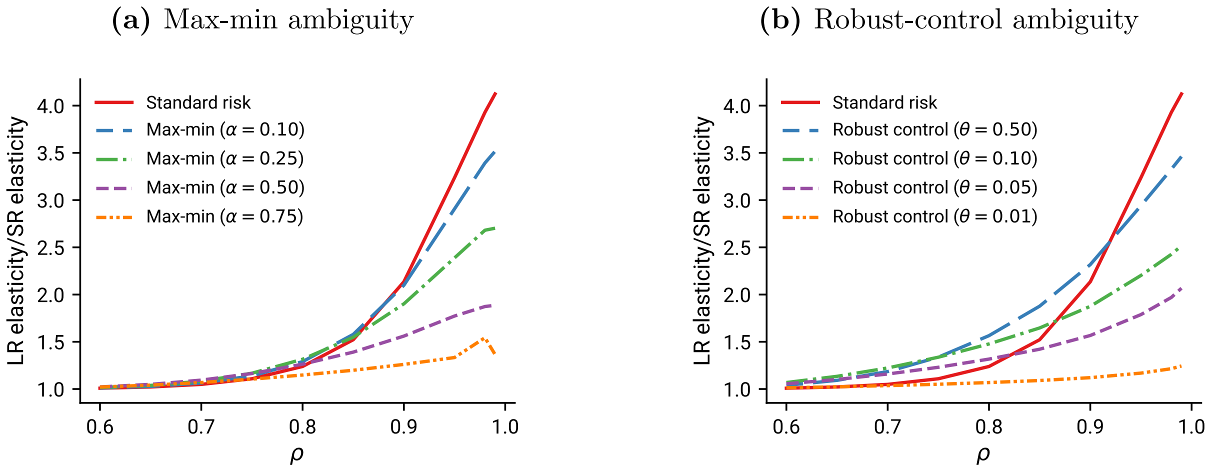

*Notes:* Shows regions where one, both, or neither of the right-hand sides of (38) and (39) are positive.

**Figure 3:** Estimated trade elasticity dynamics in model simulations




*Notes:* Figure shows estimates of trade elasticity dynamics from (55) for different levels of tariff persistence. (a), (c), (e): Max-min ambiguity for $\rho = 0.8$, $\rho = 0.95$, and $\rho = 09.99$, respectively. (b), (d), (f): Same as (a), (c), (e) but with robust-control ambiguity.

**Figure 4:** Long-run trade elasticity vs. tariff persistence in model simulations




*Notes:* Figure shows estimated long-run trade elasticity as a function of tariff persistence for varying degrees of trade-policy ambiguity. (a) Max-min ambiguity with $\alpha \in \{0, 0.1, 0.25, 0.5, 0.75\}$. (b) Robust-control ambiguity with $\theta \in \{\infty, 0.5, 0.1, 0.05, 0.01\}$. X-axis: tariff persistence $\rho$. Y-axis: ratio of long-run trade elasticity estimate $\beta_{14}^X$ to short-run trade elasticity $\beta_0^X$.

**Table 1:** Assigned parameter values

| Parameter | Meaning | Value |
|---|---|---|
| $\sigma$ | Demand elasticity | 3.17 |
| $r$ | Interest rate | 0.04 |
| $\delta_0$ | Asymptotic exit rate | 21 |
| $\delta_1$ | Elasticity of exit to productivity | 0.02 |
| $\rho_\xi$ | Variable trade cost persistence | 0.92 |
| $\kappa_0$ | Sunk entry cost | 1.57 |
| $\kappa_1$ | Fixed continuation cost | 0.657 |
| $\xi_H$ | High iceberg cost | 5.89 |
| $\sigma_z$ | Productivity shock dispersion | 1.69 |

# A Proofs

Throughout, write $\gamma \equiv \beta\delta \in (0,1)$, $A := 1-\gamma$, and $\Delta := 1-\gamma(\rho-\eta) > 0$. Let $T_s = D\tau_s^{1-\sigma}$ with $\sigma > 1$ so that $T_H < T_L$. Under SR (standard risk) and MM (max–min), the low–state threshold satisfies

$$z_L^* = \frac{\kappa}{\Phi}, \qquad \Phi = \frac{\bar{T}}{A} + \frac{T_L - \bar{T}}{\Delta}, \quad \bar{T} = p_H T_H + (1-p_H)T_L, \quad p_H = \frac{\eta}{\eta + 1 - \rho},$$

and in the MM case the same formula holds with the tilted transition matrix $P'(\alpha)$:

$$\eta' = \eta + \alpha(1-\eta), \quad \rho' = \rho + \alpha(1-\rho), \quad p'_H = \frac{\eta'}{\eta' + 1 - \rho'}, \quad \Delta'(\alpha) = 1 - \gamma\big(\rho' - \eta'\big).$$

Under RC (robust control), the quadratic representation gives

$$V_L^X(z) \;=\; b_L z - \frac{a_L}{2\theta} z^2, \qquad \frac{a_L}{2\theta}\,(z_L^*)^2 - b_L z_L^* - \kappa = 0,$$

with $b_L > 0$, $a_L > 0$, and the closed form

$$z_L^{*,\mathrm{RC}}(\theta) = \frac{b_L - \sqrt{b_L^2 - \frac{2a_L}{\theta}\,\kappa}}{a_L/\theta} = \frac{b_L - \sqrt{Q}}{a_L/\theta}.$$

## A.1 Comparative statics w.r.t. $\kappa$

**Claim (30)**. *For all three setups,* $\dfrac{\partial z_L^*}{\partial \kappa} > 0$.

*(MM/SR).* Since $z_L^* = \kappa/\Phi$ with $\Phi > 0$, we have $\partial_\kappa z_L^* = 1/\Phi > 0$. □

*(RC).* From the closed form, $\frac{\partial z_L^{*,\mathrm{RC}}}{\partial \kappa} = \frac{1}{\sqrt{Q}} \;>\; 0$, since $b_L, a_L, \theta > 0$. □

**Claim (31)**. *Ambiguity increases the sensitivity to* $\kappa$*:*

$$\frac{\partial z_L^{*,\mathrm{MM}}}{\partial \kappa} > \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \kappa},\ \alpha \in (0,1], \qquad \frac{\partial z_L^{*,\mathrm{RC}}}{\partial \kappa} > \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \kappa},\ \theta > 0.$$

*(MM vs. SR:).* $\partial_\kappa z_L^{*,\mathrm{MM}}(\alpha) = 1/\Phi(\alpha)$ and $\partial_\kappa z_L^{*,\mathrm{SR}} = 1/\Phi(0)$. Because $z_L^{*,\mathrm{MM}}(\alpha) = \kappa/\Phi(\alpha) > \kappa/\Phi(0) = z_L^{*,\mathrm{SR}}$ for every $\alpha > 0$ (max–min tilt raises the threshold), it follows that $1/\Phi(\alpha) > 1/\Phi(0)$. Hence $\partial_\kappa z_L^{*,\mathrm{MM}}(\alpha) > \partial_\kappa z_L^{*,\mathrm{SR}}$. □

*(RC vs. SR:).* $\partial_\kappa z_L^{*,\mathrm{RC}} = 1/\sqrt{Q}$ and $\partial_\kappa z_L^{*,\mathrm{SR}} = 1/b_L$. For any finite $\theta > 0$ and $\kappa > 0$, $Q = b_L^2 - \frac{2a_L}{\theta}\kappa < b_L^2$, so $\sqrt{Q} < b_L$ and thus $1/\sqrt{Q} > 1/b_L$. □

**Claim (32)**. *A higher ambiguity level strengthens the effect of $\kappa$:*

$$\frac{\partial^2 z_L^{*,\mathrm{MM}}}{\partial\alpha\,\partial\kappa} > 0 \quad ,\ \alpha \in [0,1), \qquad \frac{\partial^2 z_L^{*,\mathrm{RC}}}{\partial\theta\,\partial\kappa} < 0 \quad ,\theta > 0.$$

*(MM).* Since $z_L^{*,\mathrm{MM}}(\alpha) = \kappa/\Phi(\alpha)$,

$$\frac{\partial^2 z_L^{*,\mathrm{MM}}}{\partial\alpha\,\partial\kappa} = \frac{\partial}{\partial\alpha}\left(\frac{1}{\Phi(\alpha)}\right) = -\frac{\Phi_\alpha(\alpha)}{\Phi(\alpha)^2}.$$

But $\partial_\alpha z_L^{*,\mathrm{MM}} = -\kappa\,\Phi_\alpha/\Phi^2 > 0$ for $\alpha \in [0,1)$ (the threshold increases in $\alpha$), hence $-\Phi_\alpha/\Phi^2 > 0$. □

*(RC).* Using $\partial_\kappa z_L^{*,\mathrm{RC}} = Q^{-1/2}$ with $Q = b_L^2 - \frac{2a_L}{\theta}\kappa$,

$$\frac{\partial^2 z_L^{*,\mathrm{RC}}}{\partial\theta\,\partial\kappa} = -\frac{1}{2}Q^{-3/2}\,\frac{\partial Q}{\partial\theta} = \frac{a_L\kappa}{\theta^2\,Q^{3/2}} \ < \ 0,$$

since $a_L, \kappa, \theta, Q > 0$. □

**Claim (33)**. *Linearity (SR/MM) and convexity (RC) in $\kappa$:*

$$\frac{\partial^2 z_L^{*,\mathrm{SR}}}{\partial\kappa^2} = 0, \qquad \frac{\partial^2 z_L^{*,\mathrm{MM}}}{\partial\kappa^2} = 0, \qquad \frac{\partial^2 z_L^{*,\mathrm{RC}}}{\partial\kappa^2} > 0.$$

*(SR/MM).* $z_L^* = \kappa/\Phi$ implies $\partial^2_{\kappa\kappa} z_L^* \equiv 0$. □

*(RC).* Differentiating $\partial_\kappa z_L^{*,\mathrm{RC}} = Q^{-1/2}$ yields

$$\frac{\partial^2 z_L^{*,\mathrm{RC}}}{\partial\kappa^2} = \frac{1}{2}Q^{-3/2}\left(\frac{2a_L}{\theta}\right) = \frac{a_L/\theta}{Q^{3/2}} \ > \ 0,$$

since $a_L, \theta, Q > 0$. □

## A.2 Comparative statics w.r.t. $\tau_H$

**Claim (34).** *For each expectational setup (SR, MM, RC), the low–state entry threshold is increasing in the high tariff: $\partial z_L^*/\partial\tau_H > 0$.*

*(SR).* Write $z_L^* = \kappa/\Phi$ with

$$\Phi = \frac{\bar{T}}{1-\gamma} + \frac{T_L - \bar{T}}{\Delta}, \qquad \bar{T} = p_H T_H + (1-p_H)T_L,$$

where $T_H = D\tau_H^{1-\sigma}$ and $\sigma > 1$ so that $T_H$ is strictly *decreasing* in $\tau_H$. Since $\Delta$ and $p_H$ do not depend on $\tau_H$,

$$\frac{\partial \Phi}{\partial \tau_H} = \left(\tfrac{1}{1-\gamma} - \tfrac{1}{\Delta}\right) p_H \frac{\partial T_H}{\partial \tau_H} \ < \ 0,$$

because $\frac{1}{1-\gamma} - \frac{1}{\Delta} = \frac{\gamma(\eta+1-\rho)}{(1-\gamma)\Delta} > 0$ and $\partial T_H/\partial \tau_H < 0$. Hence $\partial_{\tau_H} z_L^* = -\kappa\, \Phi_{\tau_H}/\Phi^2 > 0$. □

*(MM).* The same calculation holds with $p_H$ and $\Delta$ replaced by the worst–case objects $p'_H(\alpha)$ and $\Delta'(\alpha)$, which also do not depend on $\tau_H$. Therefore $\Phi_{\tau_H}(\alpha) < 0$ and $\partial_{\tau_H} z_L^{*,\mathrm{MM}} > 0$. □

*(RC).* The low–state threshold satisfies $\kappa = b_L z - \frac{a_L}{2\theta} z^2$ with $b_L > 0$, $a_L > 0$. Differentiating implicitly w.r.t. $\tau_H$,

$$\left(b_L - \tfrac{a_L}{\theta} z\right) \frac{\partial z}{\partial \tau_H} = \frac{a_{L,\tau_H}}{2\theta} z^2 - b_{L,\tau_H} z.$$

Here $b_{L,\tau_H} < 0$ (because $b_L$ is linear in $T_H$, which is decreasing in $\tau_H$ with a positive weight) and $a_{L,\tau_H} > 0$ (since $a_L \propto (b_H - b_L)^2$ and $|b_H - b_L|$ increases as $T_H$ falls). The coefficient $b_L - \frac{a_L}{\theta} z = \sqrt{b_L^2 - \frac{2a_L}{\theta}\kappa} > 0$. Hence the right–hand side is positive and $\partial_{\tau_H} z_L^{*,\mathrm{RC}} > 0$. □

**Claim (35).** *Ambiguity strictly amplifies the $\tau_H$-sensitivity:*

$$\frac{\partial z_L^{*,\mathrm{MM}}}{\partial \tau_H} > \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \tau_H} \quad (\alpha > 0), \qquad \frac{\partial z_L^{*,\mathrm{RC}}}{\partial \tau_H} > \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \tau_H} \quad (\theta \in \mathbb{R}_+).$$

*MM vs. SR).* Using the SR/MM formulas,

$$\frac{\partial z_L^*}{\partial \tau_H} = \frac{\kappa}{\Phi(\cdot)^2} \left(\tfrac{1}{1-\gamma} - \tfrac{1}{\Delta(\cdot)}\right) p_H(\cdot) \left|\tfrac{\partial T_H}{\partial \tau_H}\right|,$$

where $(\cdot)$ denotes either the baseline $(p_H, \Delta)$ or the worst–case $(p'_H, \Delta')$. Hence

$$\frac{\partial_{\tau_H} z_L^{*,\mathrm{MM}}}{\partial_{\tau_H} z_L^{*,\mathrm{SR}}} = \left(\frac{\Phi}{\Phi(\alpha)}\right)^2 \frac{p'_H}{p_H} \frac{\Delta}{\Delta'}.$$

First, $\Phi(\alpha) < \Phi$ for $\alpha > 0$, so the prefactor $(\Phi/\Phi(\alpha))^2 > 1$. Second,

$$\frac{p'_H}{p_H} \cdot \frac{\Delta}{\Delta'} = \frac{\eta'}{\eta} \cdot \frac{\Delta}{\Delta'} \ \geq \ 1,$$

because $\eta' = \eta + \alpha(1-\eta)$ and $\Delta' = 1 - \gamma(\rho' - \eta') = 1 - \gamma(1-\alpha)(\rho - \eta)$ imply

$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \log\!\left(\frac{\eta'}{\Delta'}\right) = \frac{1-\eta}{\eta'} - \frac{\gamma(\rho-\eta)}{\Delta'} \ \geq \ 0 \quad \iff \quad 1 - \eta - \gamma(\rho - \eta) \geq 0,$$

which holds since $\gamma \in (0,1)$ and $\rho, \eta \in [0,1]$ (indeed $1-\eta-\gamma(\rho-\eta) = 1-\gamma\rho-(1-\gamma)\eta \geq 0$). Therefore the product above is $\geq 1$ with strict $>$ for $\alpha > 0$. Hence $\partial_{\tau_H} z_L^{*,\mathrm{MM}} > \partial_{\tau_H} z_L^{*,\mathrm{SR}}$. □

*RC vs. SR).* From the implicit formula,

$$\frac{\partial z_L^{*,\mathrm{RC}}}{\partial \tau_H} = \frac{-\,b_{L,\tau_H}\,z}{\sqrt{Q}} + \frac{a_{L,\tau_H}}{2\theta}\,\frac{z^2}{\sqrt{Q}}, \qquad Q := b_L^2 - \frac{2a_L}{\theta}\kappa.$$

By contrast, $\dfrac{\partial z_L^{*,\mathrm{SR}}}{\partial \tau_H} = -\dfrac{\kappa\, b_{L,\tau_H}}{b_L^2}$. Subtracting and using $\kappa = b_L z - \frac{a_L}{2\theta} z^2$,

$$\frac{\partial z_L^{*,\mathrm{RC}}}{\partial \tau_H} - \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \tau_H} = \Big(-\,b_{L,\tau_H}\Big)\left[\frac{z}{\sqrt{Q}} - \frac{\kappa}{b_L^2}\right] + \frac{a_{L,\tau_H}}{2\theta}\,\frac{z^2}{\sqrt{Q}}.$$

Since $b_{L,\tau_H} < 0$, $a_{L,\tau_H} > 0$, and $\sqrt{Q} < b_L$,

$$\frac{z}{\sqrt{Q}} - \frac{\kappa}{b_L^2} = z\Big(\frac{1}{\sqrt{Q}} - \frac{1}{b_L}\Big) + \frac{a_L z^2}{2\theta\, b_L^2} \;>\; 0,$$

so both terms on the right are strictly positive. Hence $\partial_{\tau_H} z_L^{*,\mathrm{RC}} > \partial_{\tau_H} z_L^{*,\mathrm{SR}}$ for all $\theta > 0$. $\square$

**Claim (36).** *Cross–effects with the ambiguity parameter.*

$$\frac{\partial^2 z_L^{*,\mathrm{MM}}}{\partial\alpha\,\partial\tau_H} > 0 \quad (\alpha\in[0,1)), \qquad \frac{\partial^2 z_L^{*,\mathrm{RC}}}{\partial\theta\,\partial\tau_H} < 0 \quad (\theta>0).$$

*(MM).* As shown above,

$$\frac{\partial z_L^{*,\mathrm{MM}}}{\partial \tau_H} = \frac{\kappa\,\gamma}{1-\gamma}\,\frac{\eta'}{\Delta'\,\Phi(\alpha)^2}\left|\tfrac{\partial T_H}{\partial \tau_H}\right|.$$

The factor $\eta'/\Delta'$ is weakly increasing in $\alpha$ (same inequality as in the proof of (35)), while $\Phi(\alpha)$ is strictly decreasing in $\alpha$; hence the whole expression is strictly increasing in $\alpha$ on $[0,1)$. $\square$

*(RC).* Using the representation $\partial_{\tau_H} z = U/\sqrt{Q}$ with

$$U(\theta) = \frac{a_{L,\tau_H}}{2\theta}\,z^2 - b_{L,\tau_H} z \quad \text{and} \quad Q(\theta) = b_L^2 - \frac{2a_L}{\theta}\kappa,$$

we have

$$\frac{\partial}{\partial\theta}\Big(\frac{\partial z}{\partial \tau_H}\Big) = \frac{U_\theta}{\sqrt{Q}} - \frac{U\,Q_\theta}{2\,Q^{3/2}}, \qquad Q_\theta = \frac{2a_L\kappa}{\theta^2} > 0.$$

Since $a_{L,\tau_H} > 0$, $-b_{L,\tau_H} > 0$, and $z_\theta < 0$,

$$U_\theta = -\frac{a_{L,\tau_H}}{2\theta^2} z^2 + \frac{a_{L,\tau_H}}{\theta} z z_\theta - b_{L,\tau_H} z_\theta \;<\; 0.$$

Thus the first term is negative and the second term is also negative (because $U > 0$, $Q_\theta > 0$).

Therefore $\partial^2 z_L^{*,\mathrm{RC}}/(\partial\theta\,\partial\tau_H) < 0$ for all $\theta > 0$. $\square$

## A.3 Comparative statics w.r.t. $\eta, \rho$

**Claim (37).** *For $p \in \{\eta, \rho\}$ and for each expectational setup (SR, MM, RC), the low–state entry threshold is increasing in $p$: $\partial z_L^*/\partial p > 0$.*

*(SR).* With $z_L^* = \kappa/\Phi$ and $\Phi = \bar{T}/(1-\gamma) + (T_L - \bar{T})/\Delta$ where $\bar{T} = p_H T_H + (1-p_H)T_L$, $p_H = \eta/(\eta+1-\rho)$, and $\Delta = 1-\gamma(\rho-\eta)$, we have

$$\Phi_p \;=\; \bar{T}_p\left(\frac{1}{1-\gamma} - \frac{1}{\Delta}\right) \;-\; (T_L - \bar{T})\frac{\Delta_p}{\Delta^2}.$$

Since $T_H < T_L$ (because $\sigma > 1$), $\bar{T}_\eta = (1-\rho)(T_H - T_L)/(\eta+1-\rho)^2 < 0$ and $\bar{T}_\rho = \eta(T_H - T_L)/(\eta+1-\rho)^2 < 0$, while $\Delta_\eta = +\gamma$ and $\Delta_\rho = -\gamma$. Moreover $1/(1-\gamma) - 1/\Delta = \gamma(\eta+1-\rho)/[(1-\gamma)\Delta] > 0$. Hence $\Phi_p < 0$ for $p \in \{\eta, \rho\}$, and therefore

$$\frac{\partial z_L^*}{\partial p} \;=\; -\frac{\kappa\,\Phi_p}{\Phi^2} \;>\; 0.$$

$\square$

*(MM).* Under rectangular max–min, $z_L^{*,\mathrm{MM}}(\alpha) = \kappa/\Phi(\alpha)$ with the same $\Phi(\cdot)$ but evaluated at the tilted matrix $P'(\alpha)$ and, crucially,

$$\Phi_p(\alpha) = (1-\alpha)\,\mathcal{B}_p(\eta'(\alpha), \rho'(\alpha)),$$

where $\mathcal{B}_p$ is the SR bracket above. Since $\mathcal{B}_p < 0$ at any admissible $(\eta, \rho)$ and $(1-\alpha) \geq 0$, it follows that $\Phi_p(\alpha) < 0$ for $\alpha \in [0,1)$ and $\partial_p z_L^{*,\mathrm{MM}}(\alpha) > 0$. As $\alpha \uparrow 1$, $(1-\alpha) \to 0$ and $\partial_p z_L^{*,\mathrm{MM}}(\alpha) \to 0$. *(SR/MM objects as defined in §2.3–2.4.)* $\square$

*(RC).* Under robust control, the low–state threshold solves $\frac{a_L}{2\theta}z^2 - b_L z - \kappa = 0$ with $b_L > 0$, $a_L > 0$, and

$$\frac{\partial z_L^{*,\mathrm{RC}}}{\partial p} = \frac{(a_{L,p}/2\theta)\,z^2 - b_{L,p}z}{b_L - \frac{a_L}{\theta}z} = \frac{z}{\sqrt{b_L^2 - \frac{2a_L}{\theta}\kappa}}\left(\frac{a_{L,p}}{2\theta}z - b_{L,p}\right).$$

Here $b_{L,p} < 0$ for $p \in \{\eta, \rho\}$ because $b_L = (I-\gamma P)^{-1}_{L\cdot}T$ is linear in $T$ and $T_H < T_L$; the denominator is $\sqrt{b_L^2 - \frac{2a_L}{\theta}\kappa} > 0$. When $a_{L,p} \geq 0$ the numerator is strictly positive. When $a_{L,p} < 0$, one can argue directly from the robust operator that, holding $z$ fixed, increasing $\eta$ (resp. $\rho$) shifts probability mass toward the lower continuation value, which *decreases* $V_L^X(z)$

and hence raises the unique $z$ that solves the entry condition.[4] Hence $\partial_p z_L^{*,\mathrm{RC}} > 0$ for all $\theta > 0$. $\square$

**Claim (38).** *(Max–min attenuation with $\alpha$.) For $p \in \{\eta, \rho\}$,*

$$\lim_{\alpha\to 1} \frac{\partial z_L^{*,\mathrm{MM}}}{\partial p} = 0, \qquad \exists\, \bar{\alpha}_p \in [0,1) \text{ s.t. } \operatorname{sgn}\left(\frac{\partial z_L^{*,\mathrm{MM}}}{\partial p} - \frac{\partial z_L^{*,\mathrm{SR}}}{\partial p}\right) = \operatorname{sgn}(\bar{\alpha}_p - \alpha).$$

*Moreover, $\bar{\alpha}_\eta = 0$ for all admissible primitives, whereas $\bar{\alpha}_\rho$ can be $0$ or interior in $(0,1)$.*

*Proof.* From the expression $\partial_p z_L^{*,\mathrm{MM}}(\alpha) = -\kappa\, \Phi_p(\alpha)/\Phi(\alpha)^2$ with $\Phi_p(\alpha) = (1-\alpha)\mathcal{B}_p(\eta'(\alpha), \rho'(\alpha))$ and $\mathcal{B}_p < 0$, we have $\partial_p z_L^{*,\mathrm{MM}}(\alpha) > 0$ for $\alpha \in [0,1)$ and the limit to 0 as $\alpha \uparrow 1$.

Comparing to SR, define the continuous ratio

$$R_p(\alpha) := \frac{\partial_p z_L^{*,\mathrm{MM}}(\alpha)}{\partial_p z_L^{*,\mathrm{SR}}} = (1-\alpha)\left(\frac{\Phi(0)}{\Phi(\alpha)}\right)^2 \frac{|\mathcal{B}_p(\eta'(\alpha), \rho'(\alpha))|}{|\mathcal{B}_p(\eta, \rho)|}.$$

We have $R_p(0) = 1$ and $\lim_{\alpha\to 1} R_p(\alpha) = 0$, so there exists $\bar{\alpha}_p \in [0,1)$ at which $R_p(\bar{\alpha}_p) = 1$. For $p = \eta$ one can show $R'_\eta(\alpha) < 0$ on $(0,1)$: $1-\alpha$ declines, $\Phi(\alpha)$ declines, and $|\mathcal{B}_\eta(\eta'(\alpha), \rho'(\alpha))|$ also declines because $\eta'(\alpha)$ enters $\bar{T}$ and $\Delta'$ so as to reduce $|\Phi_\eta|$. Hence $R_\eta(\alpha) < 1$ for all $\alpha > 0$ and $\bar{\alpha}_\eta = 0$. For $p = \rho$, $|\mathcal{B}_\rho(\eta'(\alpha), \rho'(\alpha))|$ may initially *rise* with $\alpha$ (the persistence channel dominates) before the $(1-\alpha)$ dampening takes over, yielding either $\bar{\alpha}_\rho = 0$ or an interior $\bar{\alpha}_\rho \in (0,1)$ depending on primitives—exactly the patterns documented in the draft. $\square$

**Claim (39).** *(RC vs. SR for the $\eta$–partial.) With*

$$A_1 := (1-\gamma\rho)(1-\eta) + \gamma\rho(1-\rho), \qquad \Delta := 1 - \gamma(\rho - \eta),$$

$$\operatorname{sgn}\left(\frac{\partial z_L^{*,\mathrm{RC}}}{\partial \eta} - \frac{\partial z_L^{*,\mathrm{SR}}}{\partial \eta}\right) \;=\; \operatorname{sgn}\left(\frac{1}{\eta} - \frac{3\gamma}{\Delta} - \frac{1-\gamma\rho}{A_1}\right).$$

*Proof.* From the SR formula $z_L^{*,\mathrm{SR}} = \kappa/b_L$ we have $\partial_\eta z_L^{*,\mathrm{SR}} = -(\kappa/b_L^2)\, b_{L,\eta}$. From the RC quadratic we have

$$\partial_\eta z_L^{*,\mathrm{RC}} = \frac{(a_{L,\eta}/2\theta)\, z^2 - b_{L,\eta} z}{\sqrt{b_L^2 - \frac{2a_L}{\theta}\kappa}}.$$

[4] Formally, the mapping $W \mapsto \pi(z) + \gamma A^P[W]$ is monotone and $A^P$ is strictly decreasing in $P_{sH}$ whenever $V_H < W_L$, so the fixed point $V^X(z; P)$ is decreasing in $(\eta, \rho)$. Since $z \mapsto V_L^X(z; P)$ is strictly increasing and continuous, the solution to $V_L^X(z; P) = \kappa$ is increasing in $(\eta, \rho)$.

Subtract, substitute $\kappa = b_L z - (a_L/2\theta)z^2$, and factor out the positive term $z/[2\theta\sqrt{b_L^2 - \frac{2a_L}{\theta}\kappa}]$. After cancellations, the sign of the difference depends only on the sign of

$$a_{L,\eta} \;+\; \frac{a_L}{b_L}\big(-3\,b_{L,\eta}\big),$$

which is independent of $\theta$ and $\kappa$. Plugging the closed forms

$$b_{L,\eta} = \frac{\gamma(1-\gamma\rho)}{(1-\gamma)\Delta^2}(T_H - T_L), \qquad \frac{a_{L,\eta}}{a_L} = \frac{(1-\gamma\rho)(1-2\eta)+\gamma\rho(1-\rho)}{A_1} - \frac{3\gamma}{\Delta},$$

and using $b_L = \frac{\bar{T}}{1-\gamma} + \frac{T_L - \bar{T}}{\Delta}$ with $T_H < T_L$, one obtains (by straightforward algebra) that the sign equals

$$\operatorname{sgn}\left(\frac{1}{\eta} - \frac{3\gamma}{\Delta} - \frac{1-\gamma\rho}{A_1}\right).$$

All factors multiplying this bracket are strictly positive, so the equality of signs follows. $\square$

**Claim (40).** *(RC vs. SR for the $\rho$–partial.) With $A_1$ and $\Delta$ as above,*

$$\operatorname{sgn}\left(\frac{\partial z_L^{*,\mathrm{RC}}}{\partial\rho} - \frac{\partial z_L^{*,\mathrm{SR}}}{\partial\rho}\right) \;=\; \operatorname{sgn}\left(\frac{3}{\Delta} - \frac{2\rho-\eta}{A_1}\right).$$

*Proof of (40).* Proceeding as in the $\eta$–case and using

$$b_{L,\rho} = \frac{\gamma^2\eta}{(1-\gamma)\Delta^2}(T_H - T_L), \qquad \frac{a_{L,\rho}}{a_L} = \frac{\gamma\eta(\eta-2\rho)}{A_1} + \frac{3\gamma}{\Delta},$$

the sign of $\partial_\rho z_L^{*,\mathrm{RC}} - \partial_\rho z_L^{*,\mathrm{SR}}$ reduces to the sign of

$$a_{L,\rho} \;+\; \frac{a_L}{b_L}\big(+3\,b_{L,\rho}\big),$$

and the same positive prefactor argument delivers the claimed expression $\operatorname{sgn}\big(\frac{3}{\Delta} - \frac{2\rho-\eta}{A_1}\big)$. Again, the result is independent of $\theta$ and $\kappa$ and depends only on $(\eta,\rho)$ through $(A_1,\Delta)$. $\square$

**Corollary (no "$\eta$–only" region).** *If the right-hand side in the $\eta$–condition of Claim (39) is positive, then so is the right-hand side in the $\rho$–condition of Claim (40).*

*Proof.* First, define

$$A_1 := (1-\gamma\rho)(1-\eta) + \gamma\rho(1-\rho) > 0, \qquad \Delta := 1 - \gamma(\rho-\eta) > 0.$$

Next, define the sufficient-condition tests

$$E_\eta(\eta,\rho) := \frac{1}{\eta} - \frac{1-\gamma\rho}{A_1} - \frac{3\gamma}{\Delta}, \qquad E_\rho(\eta,\rho) := \frac{3}{\Delta} - \frac{2\rho-\eta}{A_1}.$$

$E_\eta \geq 0$ and $E_\rho \geq 0$ are sufficient for the RC partials to weakly exceed the SR partials in $\eta$ and $\rho$, respectively. These two test expressions are related to one another as follows:

$$E_\eta(\eta,\rho) \;+\; \gamma\, E_\rho(\eta,\rho) \;=\; \frac{1-2\eta-\gamma\big(\rho^2-\eta^2\big)}{\eta\, A_1}. \tag{57}$$

To see why, write out $E_\eta + \gamma E_\rho$ and collect terms over the common denominator $\eta A_1$:

$$E_\eta + \gamma E_\rho = \Big(\frac{1}{\eta} - \frac{1-\gamma\rho}{A_1} - \frac{3\gamma}{\Delta}\Big) + \gamma\Big(\frac{3}{\Delta} - \frac{2\rho-\eta}{A_1}\Big) = \frac{1-2\eta-\gamma(\rho^2-\eta^2)}{\eta A_1}.$$

The last equality follows from the definitions of $A_1$ and $\Delta$.

We want to show that it is impossible to have $E_\eta(\eta,\rho) \geq 0$ and $E_\rho(\eta,\rho) < 0$ simultaneously. Equivalently, we want to show that $E_\rho \leq 0$ implies $E_\eta < 0$. We will prove that if $E_\rho \leq 0$, then $E_\eta < 0$. From $E_\rho \leq 0$ we have

$$\frac{3}{\Delta} \;\leq\; \frac{2\rho-\eta}{A_1} \quad\Longleftrightarrow\quad 3A_1 \;\leq\; \Delta(2\rho-\eta).$$

Using $A_1 = (1-\eta) + \gamma\rho(\eta-\rho)$ and $\Delta = 1-\gamma(\rho-\eta)$, the inequality is equivalent to

$$\gamma(\rho^2-\eta^2) \;\geq\; 3-2(\rho+\eta). \tag{58}$$

Define $N(\eta,\rho) := 1-2\eta-\gamma(\rho^2-\eta^2)$. Then (58) implies

$$N(\eta,\rho) \;\leq\; 1-2\eta-\big[\,3-2(\rho+\eta)\,\big] \;=\; 2(\rho-1) \;\leq\; 0,$$

with strict $<$ whenever $\rho < 1$ (i.e., in the interior of the unit square).

By the identity (57),

$$E_\eta = \frac{N(\eta,\rho)}{\eta A_1} \;-\; \gamma\, E_\rho.$$

If $E_\rho \leq 0$, then $-\gamma E_\rho \geq 0$, while $N(\eta,\rho) \leq 0$; hence $E_\eta \leq 0$, and in fact $E_\eta < 0$ unless $\rho = 1$ and $E_\rho = 0$ (a boundary case). Thus the pair $(E_\eta \geq 0,\; E_\rho < 0)$ cannot occur.

As a check, on the locus $E_\rho = 0$ one can eliminate $\Delta$ to obtain

$$\gamma(\rho^2-\eta^2) = 3-2(\rho+\eta), \qquad \text{hence} \qquad N(\eta,\rho) = 2(\rho-1).$$

Plugging into (57) yields

$$E_\eta\Big|_{E_\rho=0} = \frac{2(\rho-1)}{\eta A_1} \;\le\; 0,$$

with strict inequality for $\rho < 1$. This confirms that the boundary $E_\rho = 0$ lies entirely in the $E_\eta \le 0$ half-plane. □

## A.4 Comparative statics w.r.t. $z$

**Claim (41)-(42).** *The one-step robust-control operator*

$$A_s^{RC}[V^X;\theta] \;=\; -\,\theta \log \sum_{s'} P_{ss'}\, e^{-V_{s'}^X/\theta},$$

*admits the entropy (Gibbs) variational representation*

$$\mathcal{A}_s^{RC}\left[V_{s'}^X;\theta\right] = \sum_{s'=L,H} \tilde{P}_{s,s'}(z;\theta) V_{s'}^X + \theta \sum_{s'=L,H} \tilde{P}_{s,s'}(z;\theta) \log\left[\frac{\tilde{P}_{s,s'(z;\theta)}}{P_{s,s'}}\right].$$

*where*

$$\tilde{P}_{s,s'}(z;\theta) = \frac{P_{s,s'}e^{-V_{s'}^X(z)/\theta}}{\sum_{r=L,H} P_{s,r}e^{-V_r^X(z)/\theta}}$$

*Proof.* The Gibbs variational formula states that for any probability vector $p$ and any vector $v$,

$$-\theta \log \sum_i p_i e^{-v_i/\theta} = \min_{q\in\Delta} \Big\{ \sum_i q_i v_i + \theta \sum_i q_i \log \frac{q_i}{p_i} \Big\}.$$

In our case, the Lagrangian for the minimization is

$$\mathcal{L}(q,\lambda) = \sum_{s'} q_{s'} V_{s'}^X(z) + \theta \sum_{s'} q_{s'} \log \frac{q_{s'}}{P_{ss'}} + \lambda\left(\sum_{s'} q_{s'} - 1\right).$$

First-order conditions give $V_{s'}^X(z) + \theta(\log q_{s'} - \log P_{ss'} + 1) + \lambda = 0$, hence $q_{s'} \propto P_{ss'}e^{-V_{s'}^X(z)/\theta}$; normalizing $\sum q_s = 1$ delivers the desired solution for $\tilde{P}$. Substituting $q = \tilde{P}_{s\cdot}(z;\theta)$ into the entropy representation gives the desired expression for $\mathcal{A}^{RC}$. □

**Claim (44).** *The robust-control tilted log-odds in (43) satisfy*

$$\frac{\partial}{\partial z} \log \frac{\tilde{P}_{s,H}(z;\theta)}{\tilde{P}_{s,L}(z;\theta)} = \frac{1}{\theta} \frac{d}{dz}\big(V_L^X(z) - V_H^X(z)\big) \;>\; 0. \tag{59}$$

*Thus the high-tariff odds are strictly increasing in productivity $z$.*

*Proof.* Differentiating (43) gives the equality above. It remains to show $\Delta'(z) := \frac{d}{dz}(V_L^X - V_H^X) > 0$. Exporter values solve

$$V^X(z) = z\,T + \gamma\,A\big(V^X(z)\big)\,, \qquad T = \begin{bmatrix} T_L \\ T_H \end{bmatrix},\ \gamma = \beta\delta \in (0,1).$$

Differentiate w.r.t. $z$:

$$\big(I - \gamma J(z)\big)\,V^{X\,\prime}(z) = T, \qquad J_{ss'}(z) := \frac{\partial A_s}{\partial V_{s'}^X}\big(V^X(z)\big).$$

The previous claim gives $J_{ss'}(z) = \tilde{P}_{ss'}(z;\theta)$. Write the off-diagonals as $b := J_{LH}(z)$ and $c := J_{HL}(z)$, so $J = \left[\begin{smallmatrix} 1-b & b \\ c & 1-c \end{smallmatrix}\right]$. Then $V' = (V_L', V_H')^\top$ solves

$$\begin{aligned} \big(1 - \gamma(1-b)\big)V_L' - \gamma b\,V_H' &= T_L, \\ -\gamma c\,V_L' + \big(1 - \gamma(1-c)\big)V_H' &= T_H. \end{aligned}$$

Subtracting the two equations and solving for $\Delta'(z) = V_L' - V_H'$ yields

$$\Delta'(z) = \frac{(1-\gamma)(T_L - T_H)}{(1-\gamma)^2 + \gamma(1-\gamma)(b+c)} = \frac{T_L - T_H}{(1-\gamma) + \gamma(b+c)}\ >\ 0,$$

because $T_L > T_H$ and $b, c \in [0,1]$. Hence $\Delta'(z) > 0$ and the claimed inequality in follows. $\square$