Real exchange rate undervaluation, financial development and growth

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Abstract

This paper studies the hypothesis that real exchange rate undervaluation can alleviate the economic symptoms of financial underdevelopment, acting as a temporary substitute for institutional reform. This hypothesis is motivated by recent empirical studies that document a link between real exchange rate undervaluation and increased growth in GDP per capita in developing countries. As further motivation I present new evidence that this effect is driven by an interaction between undervaluation and financial frictions. Using panel data on value added in manufacturing sectors at the 3-digit ISIC level for 103 countries, I find that for countries with low levels of financial development, real exchange rate undervaluation is associated with stronger growth in sectors that depend more heavily on external financing. To establish a causal relationship between undervaluation, financial development and growth and evaluate its quantitative implications I build a multi-sector semi-small open economy model with limited enforcement of financial contracts. Qualitative partial equilibrium results indicate that a government policy of subsidizing the purchase of tradeable goods undervalues the real exchange rate and loosens enforcement constraints, leading to temporary increased growth on the transition to a new steady state with higher output. The magnitude of this effect is increasing in the severity of the enforcement problem. For economies with severe enforcement problems this policy increases consumption although the quantitative effect is quite small.

1 Introduction

Financial market imperfections are widely accepted to play a significant role in cross-country differences in economic outcomes in both academic and policy circles. Reforming institutional problems like creditor protection and accounting standards are common development policy prescriptions, but these kinds of reforms tend to be difficult to implement because institutional problems are often linked to the foundations of a country's legal system (La Porta et al., 1998). This paper's hypothesis is that government policy that intentionally undervalues the real exchange rate can act as a substitute for institutional reform, alleviating the symptoms of weak financial development. This idea is motivated by the recent observation by Rodrik (2008) that real exchange rate undervaluation is systematically associated with increased growth in real GDP per capita, with a particularly strong relationship in developing countries. Toward the end of evaluating and studying the implications of this hypothesis the paper makes two main contributions. First, I provide new empirical evidence on the channel through which undervaluation stimulates growth. In particular, I show

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that the aggregate growth effect of undervaluation is stronger for countries with poorly-developed financial systems and that undervaluation stimulates stronger growth in sectors that depend more heavily on external financing. Second, I build a multi-sector model with limited enforcement of financial contracts to highlight the causal mechanism at work and assess the impact of government policy that induces undervaluation (a subsidy to the purchase of imported and domestically produced tradeable goods). In the model, the limited enforcement problem creates a financial friction that negatively affects aggregate and sector-level outcomes, with heavily externally dependent sectors hit the hardest. Real exchange rate undervaluation induced by government policy (a subsidy to the purchase of imported and domestic tradeable goods) stimulates growth, especially in externally dependent sectors. However, the quantitative effects of such a policy are small and the welfare impact is likely to be negligible.[1]

While measures of financial development tend to be strongly correlated with measures of economic development, the question of whether financial development exerts a causal influence on economic development is a controversial topic in the literature. However, there is a growing body of empirical work that demonstrates that such a causal relationship does indeed exist. Levine (2005) reviews the empirical evidence for this causality and concludes that "the preponderance of evidence suggests that both financial intermediaries and markets matter for growth even when controlling for potential simultaneity bias." Examples of such research are King and Levine (1993) and Levine et al. (2000). There are a number of papers that look at empirical evidence on the relationship between financial development and growth at more disaggregated levels. In a seminal paper, Rajan and Zingales (1998) provide evidence that sectors differ in technological dependence on external financing and that more heavily dependent sectors grow slower in countries with poorly developed financial systems. A complementary paper is Beck et al. (2008) who show that sectors that tend to have larger shares of small firms grow more slowly in developing countries. Banerjee and Duflo (2005) review the micro-level evidence on credit constraints and their effects on economic outcomes in developing countries.

In addition to these empirical studies there are a number of recent papers that use theoretical and quantitative models to assess the impact of counterfactual improvements in financial development on economic outcomes. See for example: Jeong and Townsend (2007); Matsuyama (2008); Greenwood et al. (2010); Amaral and Quintin (2010); Erosa and Cabrillana (2008); Midrigan and Xu (2010); Buera et al. (2009). Many of these papers report large aggregate effects from improvements in financial development. One recent example which highlights the importance of modeling the effects of financial development on resource allocation both across and within sectors is Buera et al. (2009). They find that across-sector and within-sector effects of financial frictions are complementary in producing large aggregate effects. My paper contributes to this literature by highlighting the importance of the interaction between financial frictions and relative prices

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[1]A transition analysis which I have yet to perform is needed to conclusively determine the welfare impact.

and the opportunity for price-distorting government policy to improve economic outcomes.

While Rodrik (2008) is the first paper to my knowledge that demonstrates a systematic link between undervaluation and growth, there are a number of other papers that study the effects of real exchange rate movements on economic aggregates and the extent to which these effects vary with a country's level of development. An empirical study in this vein which is closely related to the paper at hand is Aghion et al. (2009) which demonstrates a negative effect of real exchange rate volatility on growth, especially for countries with low levels of financial development. The key difference between that paper and my own is that my paper studies the effects of increases in real exchange rates on sectors that have large investment financing needs, while Aghion et al. (2009) focuses on the detrimental effects of large real exchange rate movements (in either direction) on sectors that have large short-term liquidity needs. Kappler et al. (2011) show that large exchange rate appreciations are associated with large drops in output and investment in developing countries, while developed countries suffer only mild drops. Korinek and Serven (2011) treat the relationship between real exchange rate undervaluation and growth from a theoretical perspective. They build a model in which undervaluation stimulates growth through a learning-by-doing externality that disproportionately affects the tradeable sector. My paper contributes to this literature by providing evidence on the channel through which undervaluation affects growth, namely that undervaluation alleviates the symptoms of frictions related to weak financial development. More generally, my paper makes a key point about the nonlinearity of the effect of relative price changes on economic development: relative price movements can have significant effects on aggregate outcomes in the presence of market frictions.

The rest of the paper proceeds as follows. Section 2 presents empirical evidence on the relationship between undervaluation and growth, focusing on two key interactions: the interaction between undervaluation and financial development and the interaction between undervaluation and external dependence. Section 3 presents the model and several qualitative partial equilibrium results that provide helpful insight into the way the model works. Section 4 presents the model's calibration and the results of several quantitative exercises. Section 5 concludes.

2   Empirical evidence

The empirical evidence in this paper builds heavily on Rodrik (2008) and Rajan and Zingales (1998). Using Rodrik's measure of undervaluation I begin by demonstrating that the interaction between financial development (measured as the ratio of private credit to GDP) and real exchange rate undervaluation has a negative effect on growth in real GDP per capita. This means that the growth effect of undervaluation is stronger for countries with poorly developed financial systems and becomes weaker as financial development increases.

This result is not at all surprising since Rodrik (2008) observes a similar effect of the interaction between undervaluation and GDP per capita, and financial development and GDP per capita are highly correlated. So while this is not a particularly novel result, it is the first piece of evidence that financial development plays a role in the effect of undervaluation on growth.

With the link between financial development and undervaluation established at the aggregate level I move on to looking at growth at the level of manufacturing sectors at the 3-digit ISIC level. First, I demonstrate that the total share of manufacturing value added held by sectors with high external dependence (as measured by Rajan and Zingales (1998)) is increasing in financial development. In other words, high dependence sectors tend to have smaller shares of total manufacturing in countries with weak financial systems. After establishing this symptom of poor financial development, I show that undervaluation tends to alleviate it. I do this in two ways. First, I show that undervaluation is associated with growth in average external dependence of the manufacturing sector. This suggests that undervaluation causes sectors with high external dependence to grow more than sectors with low external dependence. Second, I demonstrate that this is indeed the case. Looking at growth rates at the 3-digit sector level, there is a positive interaction between undervaluation and external dependence in countries in the bottom half of the financial development distribution.

2.1 Data

The empirical results in this paper make use of several data sets as well as calculations by other authors. Macroeconomic variables (real exchange rate, GDP per capita, trade openness, inflation, etc.) come from the Penn World Tables 6.3 and the World Bank's World Development Indicators database. Financial development is measured as the ratio of private credit to GDP. This is taken from Beck et al. (2009). Data on manufacturing sectors at the 3-digit ISIC level comes from the 2005 Industrial Statistics Yearbook published by the UN Industrial Development Organization. As is standard in the growth literature, I use averages over non-overlapping five-year periods to filter out the effects of short-term fluctuations. In several exercises I split the data into two subsamples: developing countries and developed countries. My measure of financial development, denoted FD, is approximately log-normally distributed, so simply I take the developing country subsample to be the set of country-time observations with FD_{it} less than $\text{mean}_{i,t}\{\log FD_{it}\}$, where i and t index country and time respectively. The developed country subsample is the complement. Figure 1 depicts this division graphically.

Figure 1: Determination of developing and developed country subsamples
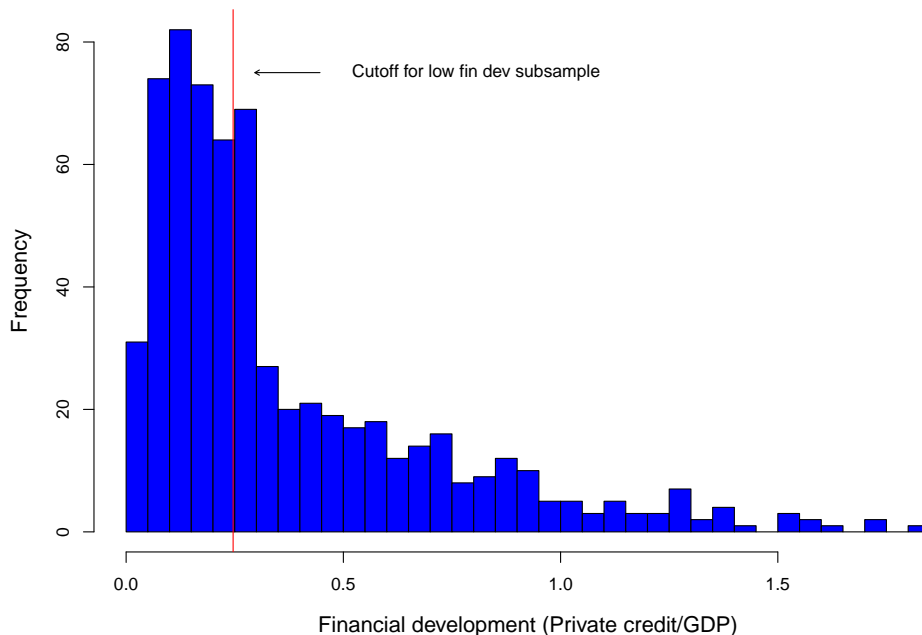
To construct a measure of real exchange rate undervaluation I follow Rodrik (2008). Loosely, undervaluation is measured as the real exchange rate adjusted for the Balassa-Samuelsson effect. More precisely, I first regress the log of the real exchange rate on the log of real GDP per capita to obtain an estimate of the Balassa-Samuelsson effect:

$$\log RER_{i,t} = \alpha + \beta \log GDP_{i,t} + f_t + \epsilon_{i,t} \tag{1}$$

where f_t is a time fixed effect. This specification requires that real exchange rate levels be comparable across countries. The inverse of the variable p (the price level of gross domestic product) from the Penn World Tables fills this need. Denote the predicted real exchange rate by $\widehat{RER}_{i,t}$. These predicted values are intended to serve as a proxy for a country's "natural" real exchange rate. I then calculate undervaluation as

$$\log UNDERVAL_{i,t} = \log RER_{i,t} - \log \widehat{RER}_{i,t}. \tag{2}$$

In words, undervaluation is the percent difference between the actual real exchange rate and the natural real exchange rate. This is precisely the same definition of undervaluation that I will use in the model of section 3. However, in the model there will be no need to estimate a proxy for the natural real exchange rate; the natural rate will simply be the prevailing equilibrium real exchange rate absent price-distorting government

intervention.

Finally, for external dependence of 3-digit ISIC level manufacturing sectors I rely on the measure constructed by Rajan and Zingales (1998). Calculated from US data, this measure is intended to capture the extent to which firms in different manufacturing sectors would rely on external financing if they were operating in an economy with minimal financial frictions. They define a sector's external dependence as the median across sampled firms in the sector of the fraction of capital expenditures that cannot be financed by operating cash flows. Sectors that have high external dependence measures like plastic products (ISIC code 356) and electric machinery (ISIC code 383) rely heavily on external finance in the US, while sectors like tobacco (314) and pottery (361) do not. A key idea in both this paper and Rajan and Zingales (1998) is that sectors that depend more heavily on external finance will be more constrained in financially underdeveloped countries. Rajan and Zingales showed that heavily dependence sectors tend to grow more slowly in financially underdeveloped countries. The paper at hand suggests that real exchange rate undervaluation alleviates the frictions that drive their result, and studies the extent to which governments of developing countries can implement policy that takes advantage of this relationship. This external dependence measure has been used in a number of other studies. For example, Beck (2003) shows that countries with poorly-developed financial systems have lower export shares and trade balances in externally-dependent sectors. Buera et al. (2009) construct external dependence measures using the Rajan and Zingales (1998) method for services and manufacturing as a whole and find that the external dependence of manufacturing is more than twice that of services. I will use these calculations in the quantitative exercises later in the paper as proxies for the external dependence of tradeables and nontradeables.

2.2 The interaction between undervaluation and financial development on aggregate growth

To test the hypothesis that real exchange rate undervaluation is associated with increased growth in real GDP per capita in countries with low levels of financial development, I estimate several equations of the form

$$\Delta Y_{i,t} = \alpha + \beta Y_{i,t-1} + \gamma FD_{i,t-1} + \delta U_{i,t} + \zeta F_{i,t-1} * U_{i,t} + \eta Z_{i,t} + f_i + f_t + \epsilon_{i,t} \tag{3}$$

where i denotes the country index and t denotes the current time period. The independent variables are: $Y_{i,t-1}$ (lagged log real GDP per capita); $FD_{i,t-1}$ (lagged financial development); $U_{i,t}$ (log undervaluation); $Z_{i,t}$ (a vector of controls); and f_i and f_t (country and time fixed effects). The dependent variable $\Delta Y_{i,t}$ is annualized growth in real GDP per capita between period $t-1$ and period t. Table 1 contains the results for this specification. Column 1 contains coefficient estimates for the entire sample of countries. The estimates

for the coefficient on undervaluation is positive and significant, indicating that undervaluation is associated with increased growth in real GDP per capita. The estimate is close to the one reported by Rodrik (2008). The coefficient on the interaction term is negative and significant, which indicates that the growth effect of undervaluation is stronger for countries with low levels of financial development. To get a sense of the economic magnitude of these estimates we can calculate the effect of a one-standard deviation increase in undervaluation for countries at different points in the distribution of FD. For a country like Uganda in 2003 at the 5th percentile of $\log FD$, the growth effect is 0.72% per year. For a country at the 25th percentile like Malaysia in 1968, the effect is 0.66% per year. For the Philippines in 1988 (50th percentile), Indonesia in 1998 (75th percentile), Singapore in 1998 (95th percentile) the growth effects are 0.57%, 0.42% and 0.02% respectively.

This result suggests that the economic channel through which undervaluation stimulates growth is only present in countries with weak financial systems. To see this another way, I estimate the same specification without the interaction term for the developing and developed country samples separately. Columns 2 and 3 list coefficient estimates for these two samnples. For the developing country sample the coefficient on undervaluation is positive and significant, while it is negative and not significantly for the developed country sample. This provides additional evidence that the growth effect of undervaluation is varies with financial development.

Table 1: Growth effects of real exchange rate undervaluation on real GDP per capita

Coefficient	(1)	(2)	(3)
Log initial GDP per capita	-0.0369	-0.0463	-0.0542
	$(0.0050)^{**}$	$(0.0869)^{**}$	$(0.0064)^{**}$
Initial financial development	-0.0047	0.0328	-0.0090
	(0.0069)	(0.0407)	(0.0062)
Log undervaluation	0.0197	0.0317	-0.0053
	$(0.0053)^{**}$	$(0.0067)^{**}$	(0.0073)
Underval. * fin. dev.	-0.0173	-	-
	$(0.0084)^{*}$		
Country fixed effects	yes	yes	yes
Time fixed effects	yes	yes	yes
Observations	501	251	250

Robust standard errors in parentheses. Controls are inflation, trade openness and government spending/GDP.
[*] Significant at 5% level.
[**] Significant at 1% level.

2.3   Average external dependence of the manufacturing sector

To provide evidence that undervaluation stimulates growth in developing countries by alleviating the effects of financial frictions, I turn to data on value added in manufacturing sectors at the 3-digit ISIC level. Rajan and Zingales (1998) show that sectors that depend more heavily on external financing grow more slowly in countries with weak financial systems. My view is that this result highlights one of the economic symptoms of poor financial development. I will address the implications of undervaluation for this symptom in more detail shortly, but first I consider a more basic symptom: sectors that depend more heavily on external finance tend to have smaller shares of overall value added in manufacturing in developing countries. To make this point, I use the external dependence measure calculated by Rajan and Zingales (1998) calculate the average external dependence of overall manufacturing, weighted by the value added of the component sectors. Figure 2 shows a strong positive relationship between financial development and average external dependence. This suggests that the financial frictions associated with low financial development constrain heavily dependent sectors more severely.

Figure 2: Financial development and average external dependence

Financial development vs. average external dependence (weighted by value added)
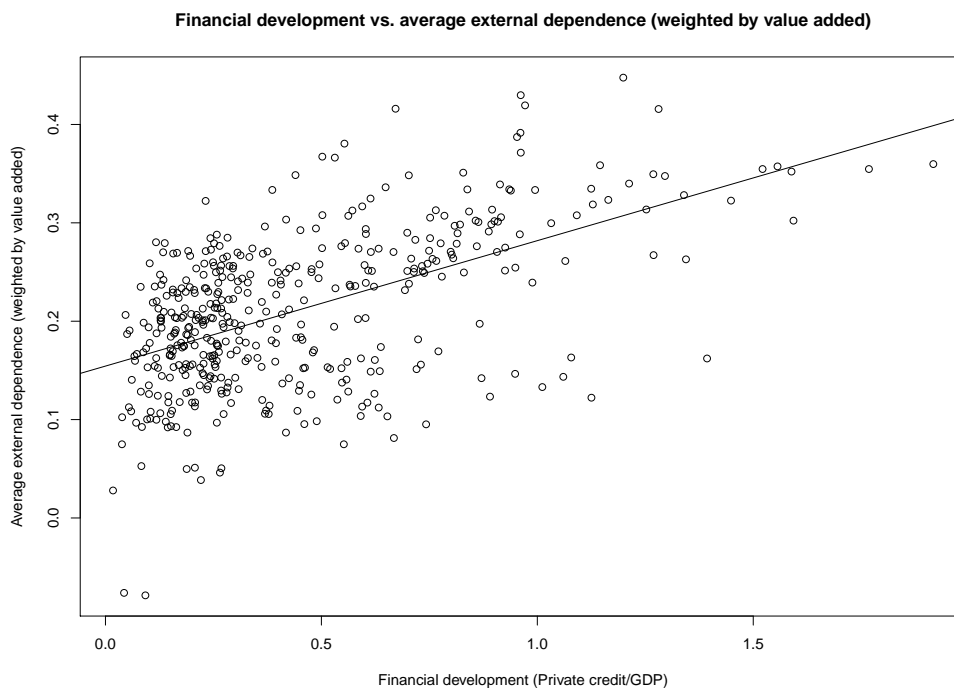
Having argued that low average external dependence in manufacturing is a symptom of low financial development, I now test the hypothesis that undervaluation alleviates this symptom. I estimate variants of the following equation:

$$\Delta A_{i,t} = \alpha + \beta Y_{i,t-1} + \gamma FD_{i,t-1} + \lambda A_{i,t-1} + \delta U_{i,t} + \zeta F_{i,t-1} * U_{i,t} + \eta Z_{i,t} + f_i + f_t + \epsilon_{i,t} \qquad (4)$$

This specification is similar to the one in (3). The additional independent variable $A_{i,t-1}$ is lagged external dependence. The dependent variable $\Delta A_{i,t}$ is the annualized change in average external dependence.[2].

Table 2 presents the results.

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[2]I do not take logs of average external dependence because the Rajan and Zingales (1998) measure is already a percentage. The dependent variable $\Delta A_{i,t}$ is the annualized change in this percentage.

Table 2: Growth effects of real exchange rate undervaluation on average external dependence of the manufacturing sector

Coefficient	(1)	(2)	(3)
Log initial GDP per capita	0.0022	0.0012	0.0073
	(0.0018)	(0.0023)	$(0.0025)^{**}$
Initial financial development	0.0022	0.0084	0.0057
	(0.0025)	(0.0091)	$(0.0021)^{**}$
Initial average external dependence	-0.1117	-0.0907	-0.1516
	$(0.0096)^{**}$	$(0.0162)^{**}$	$(0.0117)^{**}$
Log undervaluation	0.0047	0.0076	-0.0011
	$(0.0019)^{**}$	$(0.0016)^{**}$	(0.0022)
Underval. * fin. dev.	-0.0070	-	-
	$(0.0030)^{*}$		
Country fixed effects	yes	yes	yes
Time fixed effects	yes	yes	yes
Observations	268	105	163

Robust standard errors in parentheses. Controls are inflation, trade openness and government spending/GDP.
* Significant at 5% level.
** Significant at 1% level.

The three columns are analogous to the ones in table 1. The first column contains coefficient estimates for the entire sample. Just like in the estimation of growth in real GDP per capita, the coefficient on undervaluation in this specification is positive while the coefficient on the interaction term is negative (both are significant). This indicating that undervaluation is associated with increased growth in average external dependence, but this effect decreases with financial development. Given the view that figure 2 represents a symptom of weak financial development, the interpretation of these results is consistent with the paper's hypothesis: undervaluation alleviates the symptom. The effect decreases with financial development because this symptom is not present in countries with well-developed financial systems; there is nothing to alleviate. Using the same countries at different percentiles of the FD distribution as before, the effects of a one-standard deviation increase in undervaluation on annualized growth in average external dependence are: Uganda-2003 (5th percentile) 0.16%; Malaysia-1968 (25th percentile) 0.14%; Philippines-1988 (50th percentile) 0.11%; Indonesia-1998 (75th percentile) 0.04%; Singapore-1998 (95th percentile) -0.1%.

The second and third columns contain the estimates for the developing and developed country subsam-

ples. As before, we see that the coefficient on undervaluation is positive and significant for the developing country sample while it is negative and insignificant for the developed country sample. This shows again that the effect of undervaluation on average external dependence differs with financial development.

2.4 Growth at the level of 3-digit manufacturing sectors

As a further test of the theory that undervaluation drives growth by alleviating financial frictions, I look at the effect of undervaluation on growth in individual manufacturing sectors at the 3-digit ISIC level. In particular, we are interested in the growth effect of the interaction between undervaluation and a sector's external dependence, and how this effect varies with financial development at the country level. Because interpreting three-way interaction terms is difficult, I estimate equations of the form

$$\Delta Y_{i,j,t} = \alpha + \beta S_{i,j,t-1} + \delta U_{i,t} + \zeta E_j * U_{i,t} + \eta Z_{i,t} + f + f_i + f_t + \epsilon_{i,j,t} \tag{5}$$

for the developing and developed country samples separately. Here i and t still index country and time, while j indexes sector. $S_{i,j,t-1}$ is the lag of sector j's share of overall manufacturing value added in country i. E_j is the sector's external dependence. The vector of controls $Z_{i,t}$ is the same as in the previous specifications. The dependent variable $\Delta Y_{i,j,t}$ is annualized growth in real value added at the sector level. This specification is similar to the one in Rajan and Zingales (1998), except that undervaluation rather than financial development is interacted with external dependence.

Table 3 present the results for this specification. The first column contains the estimates for the sample of developing countries while the second column contains the results for the developed country sample.

Table 3: Growth effects of real exchange rate undervaluation on real value added for 3-digit manufacturing sectors

Coefficient	(1)	(2)
Initial sector share	-0.0163	0.0254
	(0.0187)	(0.0196)
Log undervaluation	0.1543	0.0693
	(0.0106)**	(0.0097)**
Underval. * external dependence	0.0267	-0.0147
	(0.0064)**	(0.0053)**
Country fixed effects	yes	yes
Sector fixed effects	yes	yes
Time fixed effects	yes	yes
Observations	1373	1444

Robust standard errors in parentheses. Coefficients and standard errors for fixed effects not reported.
[*] Significant at 5% level.
[**] Significant at 1% level.

While the estimates of δ for both samples are positive, the estimate for the developing country sample is much larger. More importantly, the estimate of ζ (the coefficient on the interaction term) is positive for the developing country sample and negative for the developed country sample. Both estimates are significant at the 1% level. This indicates that the effect of interaction between undervaluation and external dependence on sector-level growth differs substantially with a country's financial development. These results present additional evidence that corroborates the results for growth in average external dependence.

To get a sense of the economic significance of the estimates in table 3, consider the industry at the 25th percentile of external dependence, petroleum refineries (ISIC code 369, external dependence of 0.06), and the industry at the 75th percentile, textiles (321, 0.40). Column (1) indicates that for a country in the developing sample, a one-standard deviation increase in log undervaluation increases the annual growth rate of real value added in textiles and refineries by 6.8% and 6.4% respectively. Thus textiles will grow 0.4% per year faster than refineries. Conversely, column (2) indicates that for a country in the developed country sample, textiles will grow 0.15% per year slower than refineries in response to a one-standard deviation increased in undervaluation.

Rajan and Zingales (1998) provides useful context for these results. They found that sectors that depend more heavily on external finance tend to grow more slowly in financially underdeveloped countries. This

suggests that financial frictions associated with weak financial development are most burdensome for heavily externally dependent sectors. My results indicate that heavily dependent sectors respond more strongly to undervaluation in countries with weak financial systems. This suggests that financial constraints loosen more for heavily dependent sectors in response to undervaluation. Thus my results provide evidence that real exchange rate undervaluation stimulates growth in developing countries by alleviating the financial frictions that drive the results of Rajan and Zingales (1998). This channel will play an important role in the model of section 3 and the policy experiments of section 4.

3 Model

The value of a model in this study is twofold. Using a model to analyze the effects of government policies that affect the real exchange rate passes the Lucas critique and allows for welfare analysis while empirical study alone does not, but this is a general point and not specific to the paper at hand. More importantly, the model allows for a much more precise definition of real exchange rate undervaluation than the data allow for. In both the empirical empirical work above and in the model presented below, undervaluation is defined as the difference between the actual real exchange rate and the "natural" one. In the empirical study in the previous section, I defined the natural rate as the one implied by an estimation of the Balassa-Samuelsson effect. In the model I simply define the natural real exchange rate as the competitive equilibrium real exchange rate that would prevail in the absence of any price-distorting government policy. As we will see, this definition lets us use the model to develop intuition and qualitative results about the mechanisms through which financial frictions affect economic outcomes, how government policy can undervalue the real exchange rate, and how such policy can alleviate the frictions' effects.

In what follows, I present a model of a multi-sector semi-small open economy in the style of Kehoe and Ruhl (2009) with firm entry and exit. Firms can borrow at the exogenous world interest rate to finance growth and fixed capital expenditures required to enter and produce. However, limited enforcement of financial contracts constrains the amount that firms can borrow which gives rise to suboptimal allocation of resources. I make several assumptions which guarantee that this financial friction constrains the tradeable sector more tightly, which makes the allocation of resources in that sector less efficient relative to the nontradeable sector.

3.1 Environment

Time is discrete and there is no uncertainty. The economy is populated by a continuum of identical households. There are five goods used in production and consumption in the economy: a final good, a nontradeable

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good, a domestically produced tradeable good, and an imported tradeable good. The economy is small relative to the rest of the world but not entirely atomistic: foreigners have a downward-sloping demand function for the domestically produced tradeable good. The world interest rate r^* and the price of the imported good p_{Mt} are taken as exogenous. In the domestic intermediate production sectors, firms exit exogenously and are replaced with new firms that are born with no assets. New firms must borrow from international lenders to finance their growth. Financial contracts are not perfectly enforceable which leads to inefficient allocation of factors of production across firms.

3.1.1 Final goods producers

The final good is used for consumption, investment and for entry costs and fixed production costs paid by intermediate producers. It is produced using nontradeable goods x_{Nt}, domestically produced tradeable goods x_{Dt}, and imported tradeable goods x_{Mt} according to the nested CES production function

$$y_t = G(x_{Nt}, x_{Tt}, x_{Mt}) = \left\{ a_N x_{Nt}^\rho + a_T \left[z^T \left(b_D x_{Dt}^\eta + b_M x_{Mt}^\eta \right)^{\frac{1}{\eta}} \right]^\rho \right\}^{\frac{1}{\rho}} \tag{6}$$

where $\frac{1}{1-\rho}$ and $\frac{1}{1-\eta}$ are the elasticities of substitution between traded and nontraded goods and between domestic and foreign traded goods respectively. Final good producers are identical and perfectly competitive. Given prices p_t, p_{Nt}, p_{Dt} and p_{Mt} of the final, nontradeable, domestic tradeable and imported tradeable goods, a final good producer chooses inputs to maximize profits

$$p_t y_t - p_{Nt} x_{Nt} - \frac{1}{1+\tau} \left(p_{Dt} x_{Dt} - p_{Mt} x_{Mt} \right) \tag{7}$$

subject to (6), where τ is a subsidy to purchases of tradeable goods (imported and domestic). This implies the following marginal product pricing conditions:

$$p_t G_1(x_{Nt}, x_{Dt}, x_{Mt}) = p_{Nt} \tag{8}$$

$$p_t G_2(x_{Nt}, x_{Dt}, x_{Mt}) = p_{Dt}/(1+\tau) \tag{9}$$

$$p_t G_3(x_{Nt}, x_{Dt}, x_{Mt}) = p_{Mt}/(1+\tau) \tag{10}$$

As mentioned above, the country is small relative to the rest of the world so the price of imported tradeables p_{Mt} is taken as exogenous. For that reason, I treat this good as the numeraire throughout the rest of the paper and normalize p_{Mt} to one.

An alternative but equivalent representation of final goods production is to think of the final good as

being produced using nontradeable goods and a composite tradeable x_{Tt} produced from domestic and foreign tradeables:

$$y_t = \tilde{G}(x_{Nt}, x_{Tt}) = (a_N x_{Nt}^\rho + a_T x_{Tt}^\rho)^{\frac{1}{\rho}} \tag{11}$$

where

$$x_{Tt} = G^T(x_{Dt}, x_{Mt}) = z^T (b_D x_{Dt}^\eta + b_M x_{Mt}^\eta)^{\frac{1}{\eta}} \tag{12}$$

In this formulation, I treat the subsidy τ as an output subsidy to producers of the composite tradeable good. The equivalent profit maxization problem is then

$$\max_{x_{Nt}, x_{Tt}} p_t y_t - p_{Nt} x_{Nt} - p_{Tt} x_{Tt} \tag{13}$$

The marginal product pricing conditions under this formulation are

$$p_t \tilde{G}_1(x_{Nt}, x_{Tt}) = p_{Nt} \tag{14}$$

$$p_t \tilde{G}_2(x_{Nt}, x_{Tt}) = p_{Tt} \tag{15}$$

where p_{Tt} is given by similar marginal product pricing conditions derived from the composite tradeable producer's problem:

$$(1 + \tau) p_{Tt} G_1^T(x_{Dt}, x_{Mt}) = p_{Dt} \tag{16}$$

$$(1 + \tau) p_{Tt} G_2^T(x_{Dt}, x_{Mt}) = p_{Mt} \tag{17}$$

$$\tag{18}$$

We will see shortly why this alternative representation is useful for understanding how the real exchange rate is determined and how it responds to changes in τ.

3.1.2   Export market

The country is a semi-small open economy in the sense of Kehoe and Ruhl (2009) which means that foreign buyers of domestically produced traded goods have a downward sloping demand curve of the form

$$x_{Et} = D p_{Et}^{-\frac{1}{1-\eta}} \tag{19}$$

where x_{Et} is exports of the domestically produced tradeable, p_{Et} is the export price, and D is a parameter that reflects the relative size of the export market. This demand function implies that the rest of the world has an Armingtom aggregator for composite tradeables that is analogous to (12). Equilibrium requires that domestic tradeable producers receive the same price for goods sold at home and abroad, so p_{Et} is simply equal to p_{Dt}.

3.1.3 Real exchange rate

In its most general form, the real exchange rate in the model is simply the ratio of the foreign price level to the domestic one:

$$RER_t = \frac{p_t^*}{p_t} \tag{20}$$

Since this paper is concerned with changes in the real exchange rate rather than the level itself, we can simply normalize p_t^* and use the price series the model generates for p_t to calculate RER_t. However, to get a sense of how the real exchange rate moves in the model it is useful to use the pricing conditions for the final good producer and the composite tradeable aggregator listed above to write the real exchange rate as

$$RER_t = p_t^* \left(\frac{\tilde{G}_2(x_{Nt}, x_{Tt})}{p_{Tt}} \right) \tag{21}$$

Holding the foreign price level constant, this equation says that the real exchange rate is increasing in the marginal product of the composite tradeable in final goods production and decreasing in the price of the composite tradeable.

One kind of policy that can undervalue the real exchange rate is a simple subsidy to purchases of the imported tradeable good. Because the price of the imported intermediate good is is exogenous, subsidizing imports reduces the price of other intermediate inputs and ultimately the price of final goods. However, because the elasticity of substitution between imports and domestic tradeable goods is greater than one in the calibrated version of the model this kind of policy causes producers of the composite tradeable to substitute away from domestic tradeables towards imports. This causes reduction in the price of domestic tradeables that is large enough to outweigh any of the positive effects in that sector from undervaluation alleviating financial frictions. For this reason, I instead consider a policy of subsidizing producers of the composite tradeable good, or equivalently equally subsidizing the purchase of both domestic and imported tradeable goods. This eliminates any detrimental effect from subsitution away from domestic tradeables while still causing undervaluation. Because the elasticity of substitution between tradeables and nontradeables in the calibrated model is less than one, this policy does not create much substitution away from nontradeables.

3.1.4 Intermediate producers

Let $j \in \{N, D\}$ index the nontradeable and domestic tradeable sectors. In each period and in each sector $j = N, D$, a mass ψ of existing firms exogenously exits and a mass $1 - \psi$ of new firms enters. Intermediate producers in each sector are perfectly competitive and produce output using capital and labor rented from households according to the same decreasing returns to scale production function

$$y_j = f(k, \ell) = k^\alpha \ell^\theta \tag{22}$$

where $\alpha + \theta < 1$. The assumption of decreasing returns to scale implies that intermediate producers earn profits which they may distribute to their owners as dividends. I assume that all domestic intermediate producers are owned by domestic households. To enter the market a firm in sector j must pay an entry cost e_j in units of the final good. In addition, before beginning production in each period the firm must pay a fixed capital cost κ_j in addition to its factor rental cost $r_t k + w_t \ell$. The idea here is that at the beginning of its life the firm must purchase a factory or make another large capital expenditure of a similar sort in order to enter the market, and must pay upkeep on that factory as it depreciates throughout its life in additional to renting variable capital like machines to operate within the factory.

The purpose of these fixed costs is to differentiate the sectors in terms of dependence on external finance. In addition to reporting external dependence measures for manufacturing sectors as a whole, Rajan and Zingales (1998) also provide similar measures for young firms (firms that have been public less than ten years) and mature firms (public for at least ten years). These measures show that external finance varies with firm age in all sectors but this is especially true for sectors with high overall external dependence. The three manufacturing sectors with the highest overall external dependence for which Rajan and Zingales (1998) also provide external dependence measures for young and mature firms are radio (ISIC code 3832), office and computing (3825), and drugs (3522). The average external dependence across all radio manufacturers is 1.04. Young radio manufacturers have an average external dependence of 1.35 while for mature firms this number is only 0.39. For office and computing overall external dependence is 1.06, for young firms it is 1.16 and for mature firms it is only 0.26. For drugs overall external dependence is 1.49, for young firms it is 2.06 and for mature firms it is only 0.03. This same trend is still evident for sectors with low overall external dependence, but there is more variation in the difference between the external dependence of young and mature firms. For some sectors like pottery (ISIC code 361), leather (323), and nonmetal products (369) young firms actually have lower external dependence than mature ones, while the reverse is true for footwear (324), nonferrous metal (372), and apparel (322). The financing behavior of firms over their life cycles plays an important role in generating the model's results. We will see later on that alleviating financial frictions not only allows

middle-aged firms to produce more but also allows new firms to start larger and take less time to grow to maturity. Including both entry costs and per-period fixed costs that vary across sectors allows the model to better fit the differences over the life cycle of a firm in external dependence measures reported by Rajan and Zingales (1998) and plays a key role in generating large differences in sectoral responses to undervaluation. The parameters e_j and κ_j capture the essence of firm's life-cycle financing needs in a parsimonious way. The entry cost e_j is large relative to the per-period fixed cost κ_j, and since young firms produce less in the model this ensures that they are more externally dependent than mature firms.

A key assumption is that $e_D > e_N$ and $\kappa_D > \kappa_N$, i.e., the tradeable sector is more externally dependent than the nontradeable sector. We will see shortly that this means the tradeable sector is more constrained by the financial friction in the model and will respond more strongly to undervaluation that alleviates that friction. Because this model has only two sectors (and one of those sectors is nontradeables) the external dependence figures for manufacturing sectors at the 3-digit level reported by Rajan and Zingales (1998) don't translate directly to the model. Fortunately, Buera et al. (2009) construct external dependence measures for services and manufacturing using the same process. They find that services has an overall external dependence of 0.09 while manufacturing has an external dependence of 0.21. I use these values as proxies for the external dependence of nontradeables and tradeables in calibrating the model. External dependence in the model for an individual firm is calculated in the same way as the data analogue: the percentage of capital expenditures that cannot be financed by operating cash flows. Capital expenditures in the model for an incumbent firm producing with capital k are $r_t k + p_t \kappa_j$. An entering firm's capital expenditures are calculated the same way, with the addition of the entry cost $p_t e_j$. I define a mature firm as an incumbent firm that is operating at its unconstrained optimal scale, while young firms are new entrants and constrained firms. In the full enforcement version of the model all firms are unconstrained, so the only young firms are new entrants.

3.1.5 Financial contracts

New firms enter with no assets. I assume that they have limited liability and therefore cannot issue new equity to households. This means that they must borrow to finance the initial entry cost as well as working capital and the fixed capital cost in subsequent periods. I assume that newborn firms enter into long-term contracts with international lenders. However, these contracts are not perfectly enforceable. Firms can default, in which case they receive an outside option value $\phi(r_t k + p_t \kappa_j)$. This outside option is intended to capture the idea that the firm can back out of the long-term contract and abscond with a multiple ϕ of the short-term capital advanced by the lender. This outside option is reduced-form; it is not determined by bargaining with the lender or any other endogenous process. The parameter ϕ represents the degree to

which enforcement of financial contracts is limited in the economy. Higher values of ϕ correspond to more severe enforcement problems. In the extreme, an economy with perfect enforcement would have $\phi = 0$, while an economy in which no borrowing is possible would have $\phi = \infty$.

The contracting problem in this environment is virtually indentical to the one in Albuquerque and Hopenhayn (2004). Rather than solving the firm's problem directly we can solve the dual problem of the lender using the firm's equity as a state variable. Consider a firm in sector j born at time t. If the project is profitable (i.e. it is worth it for the intermediary to front the entry cost) an intermediary will offer the firm a contract $\{k_{jt}^s, \ell_{jt}^s, d_{jt}^s, v_{jt}^s\}_{s=0}^{\infty}$ where the superscript s denotes the firm's age and the subscript t denotes the time period. The contract specifies factor rental, dividend payments and equity in each period $t + s$ conditional on surviving to age s. We can recursively define the firm's equity value for $s \geq 0$ as

$$v_{jt}^s = d_{jt}^s + \psi q_{t+s} v_{jt}^{s+1} \tag{23}$$

where q_{t+s} is the firm's discount rate.[3] Limited liability requires that

$$d_{jt}^s \geq 0, \ \forall s \geq 0 \tag{24}$$

Since there is no uncertainty in the model any default would occur with probability one. Lenders have perfect foresight and anticipate this, so any potential contract must satisfy the following enforcement constraint

$$v_{jt}^s \geq \phi(r_{t+s} k_{jt}^s + p_{t+s} \kappa_j), \ \forall s \geq 0 \tag{25}$$

We can now see that an alternative interpretation of the outside option is that short-term debt must be less than a fraction $\frac{1}{\phi}$ of current equity. The presence of the prices r_{t+s} and p_{t+s} in the enforcement constraint create an externality in the model; private agents do not internalize the effects of their decisions on the tightness of firms' enforcement constraints. We will see later on that this creates an opportunity for price-distorting government intervention to improve the efficiency of market outcomes. In this sense, this model is an example of an economy in which the competitive equilibrium absent government intervention is not constrained efficient in the sense of Kehoe and Levine (1993). The fact that the per-period fixed cost κ_j shows up in the enforcement constraint makes the impact of the enforcement problem differ across sectors. Because the fixed cost is larger in the tradeable sector, the enforcement constraint is tighter (holding everything else constant) for tradeable producers relative to nontradeable producers.

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[3]In a stationary equilibrium, this discount rate will be equal to that of an international lender. However, this will not generally be the case in a nonstationary equilibrium as changes in the price of the final good will tilt the desired consumption path of domestic households.

Written recursively, the problem of an international lender in choosing this contract is

$$B_{jt}(v) = \max_{k,\ell,d,v'} \ p_{jt}f(k,\ell) - w_t\ell - r_tk - p_t\kappa_j - d + \psi\beta B_{jt+1}(v') \tag{26}$$

subject to

$$v = d + \psi q_t v' \tag{27}$$

$$d \geq 0 \tag{28}$$

$$v \geq \phi(r_tk + p_t\kappa_j) \tag{29}$$

where $\frac{1}{1+r^*}$ is the lender's discount rate and (27) - (29) are simply the recursive versions of (23) - (25). I include time subscripts on the lender's value function to stress that in a nonstationary equilibrium, prices will generally change over time so the lender's problem will be time-dependent. Denote the optimal policy functions (again potentially time-dependent) associated with this problem as $k_{jt}(v)$, $\ell_{jt}(v)$, $y_{jt}(v)$, $d_{jt}(v)$, and $v'_{jt}(v)$. Unlike in the sequential contract described above, here the subscript t denotes the current time period; the policy functions indicate the allocation a firm with equity v in period t will receive.

Initial debt B^0_{jt} and equity v^0_{jt} are required to fully specify the lender's problem. Lenders are perfectly competitive so they take initial equity as given; it's the "price" a lender must pay to enter into the contract. Taking v^0_{jt} as given, initial debt solves

$$B^0_{jt} = \max_{v \geq v^0_{jt}} \{B_{jt}(v) - p_te_j, 0\} \tag{30}$$

If the lender cannot obtain a present value of entering into the contract that exceeds the entry cost it does not offer any contract and the firm does not enter. Lenders do not need to pay any entry cost to gain access to the domestic financial market, so in equilibrium competition among lenders implies that the following zero profit condition must hold:

$$B^0_{jt} = \begin{cases} B^0_{jt} = p_te_j & \text{if } \max_{v \geq 0} B_{jt}(v) \geq p_te_j \\ B^0_{jt} = 0 & \text{otherwise} \end{cases} \tag{31}$$

This condition pins down initial equity v^0_{jt}. In the first case it satisfies $B_{jt}(v^0_{jt}) = p_te_j$. In the second the lender's value is always less than the entry cost regardless of the choice of initial equity so we simply have $v^0_{jt} = 0$. However, since there is no heterogeneity in productivity, equilibrium prices must be such that firms enter in both sectors. Therefore the second case is irrelevant. Once these initial conditions of the contract

are determined we can use the policy functions listed above to generate the full contract $\{k_{jt}^s, \ell_{jt}^s, d_{jt}^s, v_{jt}^s\}_{s=0}^{\infty}$.

The characterization of this problem is completely standard so I will describe it only briefly.[4] Let k_{jt}^* and ℓ_{jt}^* denote the unconstrained optimal factor inputs given prices at time t. They are the solution to the static problem

$$\max_{k,\ell} \ \{p_{jt}f(k,\ell) - w_t\ell - r_tk - p_t\kappa_j\} \tag{32}$$

Let v_{jt}^* denote the minimum level of equity required to sustain unconstrained production. It is given by

$$v_{jt}^* = \phi(r_t k_{jt}^* + p_t\kappa_j) \tag{33}$$

As we will see shortly, the firm's discount factor q_t is constant and always equal to $\frac{1}{1+r^*}$, the lender's discount factor. This means that the contracting problem is identical to the one in Albuquerque and Hopenhayn (2004). It is straightforward to show that no dividends would be distributed in period t unless it is possible for the firm's equity to reach v_{jt+1}^* in the following period, i.e., if $v \geq \psi q_t v_{jt+1}^*$, after which any dividend policy that satisfies the constraints and keeps equity above v_{jt+k}^* is Pareto optimal (with respect to the borrower and lender in the specific contract at hand only).

3.1.6 Government

I assume that the government can pursue only one kind of tax policy: subsidizing the output of producers of the composite tradeable good. The subsidy is financed by lump-sum taxes levied on households. In addition, I assume that the government's budget must satisy period-by-period budget balance. The government's budget constraint is

$$\tau_t p_{Tt} x_{Tt} = T_t \tag{34}$$

The left-hand side of the budget constraint is the government's outlays (subsidy payments) and the right-hand side is its revenues (lump-sum taxes collected from households).

3.1.7 Households

There is a large number of identical households with standard time-separable preferences

$$U\left(\{c_t\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{35}$$

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[4]See Albuquerque and Hopenhayn (2004) for details.

over infinite sequences of consumption $\{c_t\}_{t=0}^{\infty}$. The household's discount factor β is assumed to be equal to $\frac{1}{1+r^*}$. Households supply labor inelastically and rent capital to intermediate producers and purchase consumption and investment from final good producers. Like intermediate producers, households can save or borrow at the world interest rate. However, I they cannot default on any debt they incur. The budget constraint of a representative household is

$$p_t(c_t + i_t) + \frac{b_{t+1}}{1 + r^*} = w_t \bar{\ell} + r_t k_t + d_t + b_t - T_t \tag{36}$$

where i_t is investment and d_t are total dividends received from intermediate producers. Capital evolves according to the standard law of motion

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{37}$$

where δ is the depreciation rate. Taking as given initial capital stock k_0 and all future prices, the problem of a household in this economy is to choose $\{c_t, i_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to maximize (35) subject to (36) and (37). The following intertemporal first order conditions for investment and fully characterizes this problem:

$$u'(c_t) = \beta u'(c_{t+1}) \left[\frac{r_{t+1}}{p_{t+1}} + 1 - \delta \right] \tag{38}$$

$$u'(c_t) = \beta(1 + r^*) \frac{p_t}{p_{t+1}} u'(c_{t+1}) \tag{39}$$

The discount factor used to price the equity of intermediate firms is

$$q_t = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} = \frac{1}{1 + r^*} = \beta \tag{40}$$

The price of the final good enters into this expression because dividends are distributed in units of the numeraire and the household must pay the final good price p_t to convert them into consumption. We can now see that the intermediate firms' discount factor is always equal to the international lenders'.

3.1.8 Equilbrium

Definition 1. *Given initial capital stock k_0, initial distributions Ψ_{j0} over equity and productivity $j \in \{N, D\}$, and goverment policy $\{\tau_t, T_t\}_{t=0}^{\infty}$, an equilibrium in this economy is:*

(i) *prices $\{p_{Nt}, p_{Dt}, p_{Et}, p_t, r_t, w_t, q_t\}_{t=0}^{\infty}$;*

(ii) *an allocation for the final good producer $\{y_t, x_{Nt}, x_{DT}, x_{Mt}\}_{t=0}^{\infty}$;*

(iii) an allocation for foreign buyers of the domestically produced tradeable good $\{x_{Et}\}_{t=0}^{\infty}$;

(iv) lender's value functions $\{B_{jt}(v)\}_{t=0}^{\infty}$ and associated policy functions

$$\{k_{jt}(v), \ell_{jt}(v), y_{jt}(v), d_{jt}(v), v'_{jt}(v)\}_{t=0}^{\infty}$$

for $j \in \{N, D\}$;

(v) initial conditions for financial contracts $\{B_{jt}^0, v_{jt}^0\})_{t=0}^{\infty}$ for $j \in \{N, D\}$;

(vi) measures $\{\Psi_{jt}\}_{t=0}^{\infty}$ over equity for $j \in \{N, D\}$;

(vii) and an allocation for the household $\{c_t, i_t, k_{t+1}\}_{t=0}^{\infty}$

such that given prices for all $t \geq 0$:

1. *the final good producer's allocation satisfies the marginal product pricing conditions (8) - (10);*

2. *the foreign buyers' allocation satisfies the demand function (19);*

3. *the lender's policy functions satisfy the Bellman equations given by (26) and the lender's policy functions solve the associated maximization problem for each $j \in \{N, D\}$;*

4. *the initial contract conditions satisfy (30) and (31);*

5. *the measures $\{\Psi_{jt}\}_{t=0}^{\infty}$ are consistent with the policy functions, initial conditions and survival probability:*

$$\Psi_{jt+1}(V) = \psi \int \mathbb{1}_{\{v'_{jt}(v) \in V\}} \, \mathrm{d}\Psi_{jt} + (1-\psi)\mathbb{1}_{\{v_{jt}^0 \in V\}}, \ \forall(V), \ j \in \{N, D\} \tag{41}$$

6. *the household's allocation satisfies its budget constraint (36) and its Euler equation (38);*

7. *the market for final goods clears:*

$$c_t + i_t + \sum_{j=N,D} (1-\delta)\kappa_j + \delta \sum_{j=N,D} \kappa_j \mathrm{d}\Psi_{jt} = y_t \tag{42}$$

8. *the market for nontradeable goods clears:*

$$x_{Nt} = \int y_{Nt}(v) \, \mathrm{d}\Psi_{Nt} \tag{43}$$

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9. the market for domestically produced tradeable goods clears:

$$x_{Dt} + x_{Et} = \int y_{Dt}(v) \, \mathrm{d}\Psi_{Dt} \tag{44}$$

10. the markets for capital and labor clear:

$$k_t = \sum_{j=N,D} \int k_{jt}(v) \, \mathrm{d}\Psi_{jt} \tag{45}$$

$$\bar{\ell} = \sum_{j=N,D} \int \ell_{jt}(v) \, \mathrm{d}\Psi_{jt} \tag{46}$$

$$\tag{47}$$

11. the government's budget balance condition (34) is satisfied;

12. and the firm's discount factor q_t is given by (40).

Definition 2. *Given a constant government policy* (τ, T), *a stationary equilibrium in this economy is an equilibrium such that all elements listed in (i) through (vii) are constant. This requires that the time-invariant measures* Ψ_j *are fixed points of (41).*

Equilibrium implies the standard balance of payments identity:

$$CA_t + KA_t = 0 \tag{48}$$

where CA_t and KA_t are the current and capital account respectively. Each of these objects is made up of several components. We can write the current account as

$$CA_t = NX_t + r^* B_t \tag{49}$$

where NX_t is net exports and $r^* B_t$ is net foreign payments. Net exports is given simply by

$$NX_t = p_{Et} x_{Et} - p_{Mt} x_{Mt} \tag{50}$$

Net foreign payments is the product of the world interest rate and net foreign assets. The latter are given by

$$B_t = b_t - \sum_{j=N,D} \int B_{jt}(v) \, \mathrm{d}\Psi_{jt} \tag{51}$$

The sign of the summed lenders' values in this equation is reversed because a positive value of $B_{jt}(v)$ indicates that the firm is in debt to the lender. The capital account KA_t is simply given by

$$B_t - B_{t+1} \tag{52}$$

This formulation of the balance of payments condition makes it immediately obvious that in a stationary equilibrium the capital account must be zero which implies that the capital account must be zero as well. Thus the stationary equilibrium version of the balance of payments condition can be written simply as

$$NX = -r^* B \tag{53}$$

In other words, a stationary equilibrium with non-zero net exports can be sustained if net foreign assets are also non-zero. If net foreign assets are positive (negative) then net exports must be negative (positive). However, in the baseline model I impose the additional restriction that net exports are zero in a stationary equilibrium, which implies that $B = 0$, i.e.,

$$b = \sum_{j=N,D} \int B_j(v) \, \mathrm{d}\Psi_j \tag{54}$$

Thus net exports can only be nonzero during the transition path from one stationary equilibrium to another.

3.2 Partial equilibrium comparative statics

In this subsection I explore the implications of parameter and policy changes for stationary equilibria in the model ignoring general equilibrium effects and assuming equal output prices across sectors. First, I discuss how changing the severity of the enforcment problem affects firm growth paths and efficiency of resource allocation in intermediate sectors. I then show how real exchange rate undervaluation can alleviate the enforcement problem, which causes firms to enter with higher initial equity (and thus output), shortens the time it takes for new firms to become unconstrained, and improves efficiency of resource allocation within sectors.

3.2.1 Limited enforcement, external dependence and resource allocation within sectors

The first set of partial equilibrium results concerns the relationship between the enforcement parameter ϕ and average firm size, firm growth paths and allocative efficiency. Looking at a single intermediate sector $j \in \{N, D\}$ we will look at the differences between economies that are identical in all respects, including

prices, except for ϕ. As an expositional aid I will index contracting variables by ϕ, i.e., $B_j(v; \phi)$ is the lender's value in sector j in an economy with enforcement parameter ϕ. I omit time subscripts since the analysis focuses on stationary equilibria. Since prices are held constant, the unconstrained efficient factor demands (k_j^*, ℓ_j^*) are the same regardless of ϕ. Because of this, the minimum level of equity required to support unconstrained operation $v_j^*(\phi)$ is increasing in ϕ. Moreover, initial equity $v_j^0(\phi)$ is decreasing in ϕ. Since the maximum equity growth rate is also the same regardless of ϕ it takes longer for new firms to become unconstrained in high-ϕ economies than it does for firms in low-ϕ economies. This means that fewer firms survive long enough to become unconstrained in high-ϕ economies. Moreover, for a given equity level $v < v_j^*(\phi)$ the amount of capital and labor firms in high-ϕ economies can use is less than in high-ϕ economies. This means that firms are smaller along the entire growth path to maturity in low-ϕ economies. It follows the average firm size is decreasing in ϕ, implying that resources are allocated less efficiently is high-ϕ economies.

Figure 3 illustrates why initial equity is lower in high-ϕ economies. Looking at the lender's contract initialization problem (30) we can see that initial equity will always be at or above the point at which the lender's value function starts to decline. Otherwise lenders could improve their payoffs by offering more initial equity to firms. Using the zero-profit condition (31), we can see that initial equity is therefore the value at which the decreasing region of the lender's value function intersects the entry cost. Because constrained firms with the same level of equity produce less in the high-ϕ economy, lenders receive less value from lending to firms in that economy: the lender's value function in the high-ϕ economy is always below its value function in the low-ϕ economy for equity levels below v^*. This means that the equity level at which lender's value function intersects the entry cost is lower in the high-ϕ economy. A formal statement of this result follows.

Figure 3: Determination of initial equity for low and high values of ϕ
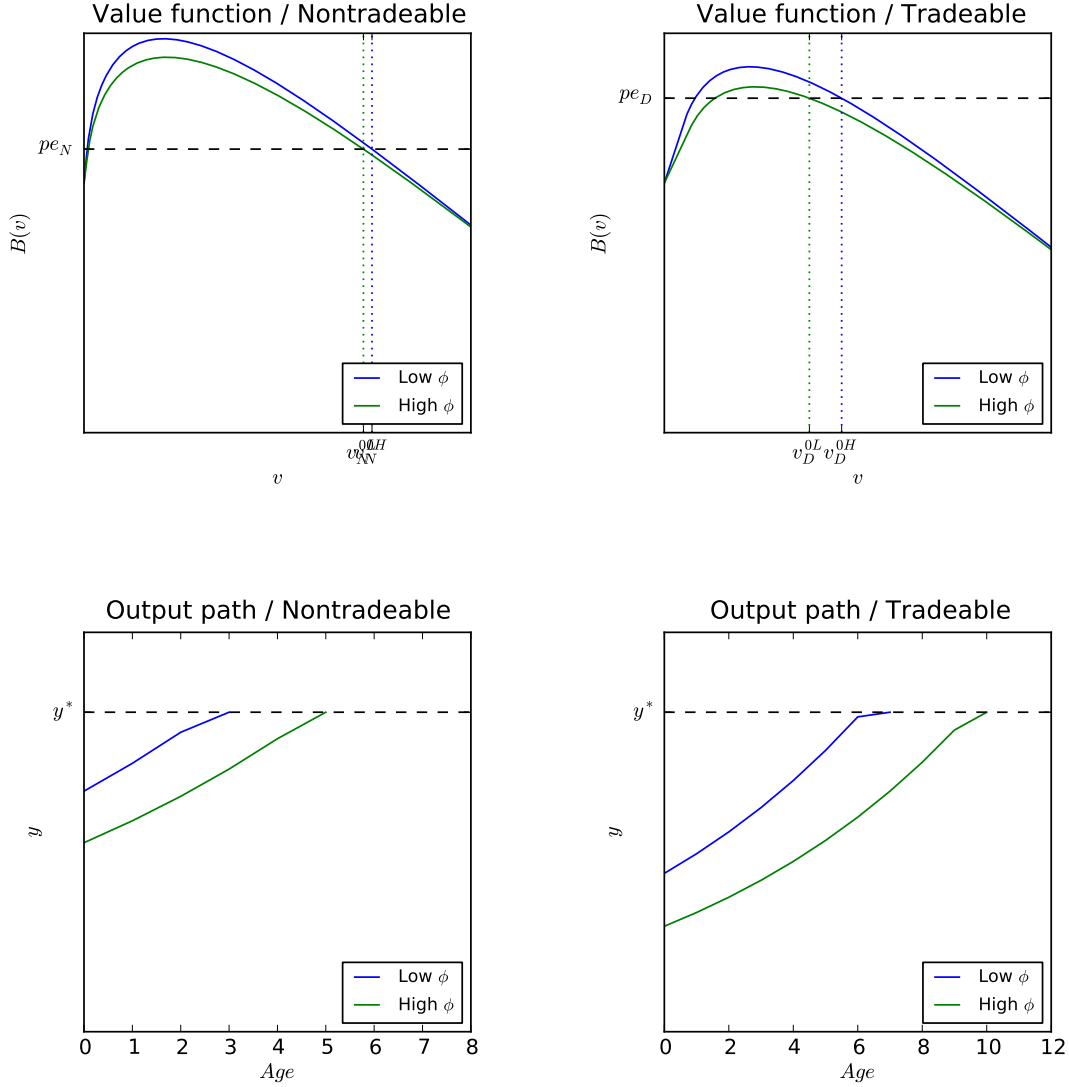
Lemma 1. *If $\phi_L < \phi_H$ then $B_j(v; \phi_H) \leq B_j(v; \phi_L)$ $\forall v$, with strict inequality $\forall v < v_j^*(\phi_H)$.*

Proposition 1. *If $\phi_L < \phi_H$ then $v_j^0(\phi_H) < v_j^0(\phi_L)$.*

The next set of partial equilibrium results compares how external dependence affects firm growth and resource allocation across sectors in the same enforcement environment. A key partial equilibrium assumption in this section is that the prices of each sector's output are equal. Thus the only difference between sectors for the sake of these results is the difference in external dependence captured by the parameter κ_j. Recall

that the tradeable sector is more externally dependent: $\kappa_D > \kappa_N$. Since output prices are the same the unconstrained factor demands are equal across sectors: $(k_N^*, \ell_N^*) = (k_D^*, \ell_D^*)$. Given this, more equity is required in the tradeable sector to sustain unconstrained operation since the term κ_j shows up in the enforcment constraint. Further, firms in the tradeable sector enter with less equity. This means it takes longer for tradeable firms to reach maturity, so fewer firms in that sector survive to become unconstrained. Constrained firms in the tradeable sector can rent less capital and labor holding equity fixed so these firms produce less over their growth paths. Thus average output and allocative efficiency in the tradeable sector is lower than in the nontradeable sector.

Figure 4 shows why initial equity is lower in the tradeable sector. The intution is similar to what happens when we vary ϕ. The lender's value is lower in the tradeable sector holding equity fixed because the enforcement constraint is tighter in the tradeable sector which leads to lower production and profit. As we saw above, this fact alone is sufficient to make initial equity lower in the tradeable sector. However, initial equity in the tradeable sector is pushed further down relative to initial equity on the nontradeable sector because the lender's value function intersects the higher entry cost at a lower equity value. As before, a formal statement of this result follows.

Figure 4: Determination of initial equity for low and high external dependence
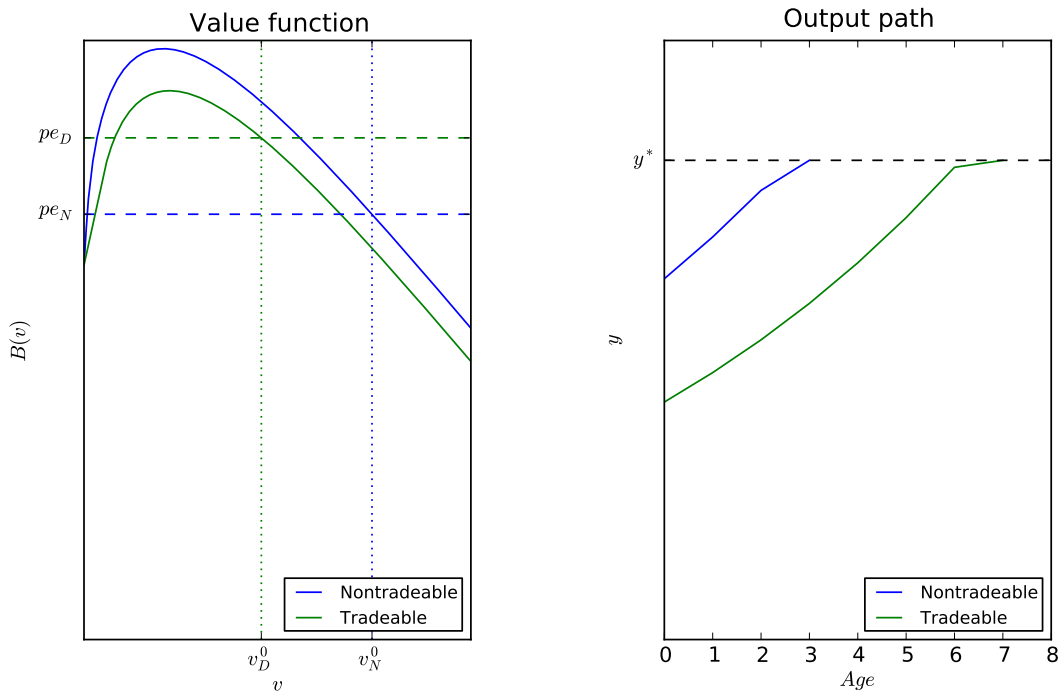
**Lemma 2.** *Assume that $p_N = p_D$. Then $B_D(v) < B_N(v)$ for all v.*

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Proposition 2. *Assume that $p_N = p_D$. Then $v_D^0 < v_N^0$.*

3.2.2 Real exchange rate undervaluation and growth

Having established the partial equilibrium effects of limited enforcement and external dependence on firm dynamics and allocation of resources within sectors, we now turn to the partial equilibrium effects of undervaluation on these same sector and firm-level outcomes. Here, I index contracting variables by p rather than ϕ. Since all other prices are fixed the unconstrained optimal factor demand does not change in response to a change in the price p of the final good. A decrease in p lowers the value of the entry cost pe_j and the per-period fixed cost $p\kappa_j$. This causes initial equity values to rise in both sectors, with the tradeable sector seeing a larger increase in initial equity because it is more externally dependent. This decreases the time it takes for firms to reach maturity so more firms survive to become unconstrained, especially in the tradeable sector. In addition, the decrease in per-period fixed cost burden allows constrained firms to rent more capital and labor and produce more on the path to maturity. Therefore average firm size increases and allocative efficiency improves, with the tradeable sector improving more.

There are two reasons that initial equity in the tradeble sector responds more to changes in the price of the final good. First, the larger change in per-period fixed cost value causes the lender's value function to increase more in the tradeable sector. This causes the intersection between the value function and the entry cost to move outward. Second, the value of the entry cost changes more for the tradeble sector which pushes this intersection point outward. Figure 5 illustrates these two mechanisms. If the external dependence of the tradeble sector is high enough, even a small change in p can lead to a large change in initial equity. This is due to concavity of the lender's value function for $v < v_j^*$. Because initial equity in the tradeble sector is closer to the maximizer of the lender's value function the slope of the value function can be very shallow at this point. This means that a small change in p (and thus $p\kappa_D$) leads to a large change in initial equity. A formal result is stated below.

Figure 5: Determination of initial equity for low and high final goods price
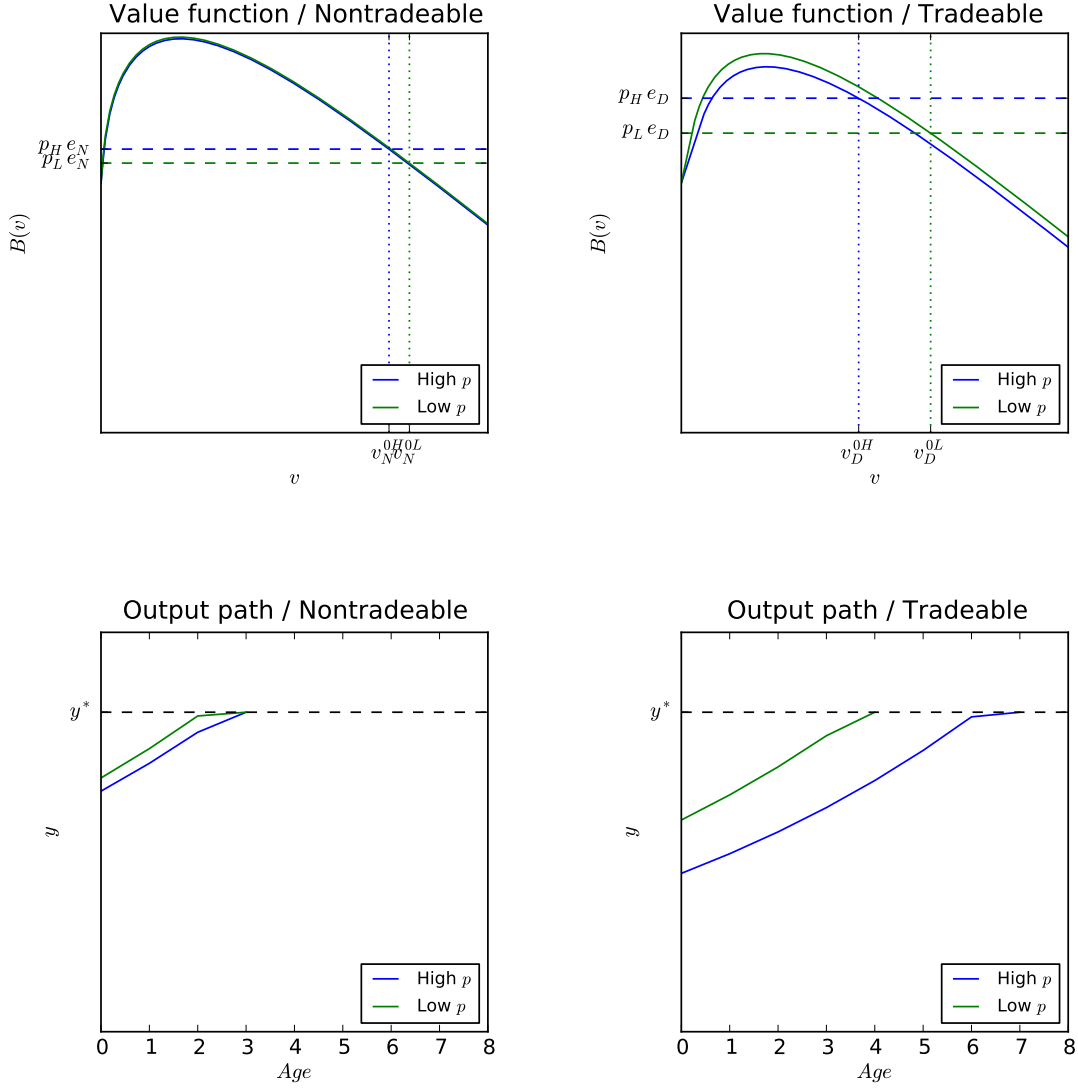
Lemma 3. *Consider a change in the price of the final good $\Delta p < 0$. In each sector j and for each v $\Delta B_j(v; p) > 0$. Moreover, $\Delta B_D(v; p) > \Delta B_N(v; p)$.*

Proposition 3. *Consider a change in the price of the final good $\Delta p < 0$. In each sector j $\Delta v_j^0(p) > 0$. Moreover, $\Delta v_D^0(p) > \Delta v_N^0(p)$.*

4 Quantitative exercises

In this section I calibrate the perfect-enforcement version of the model to U.S. data and then conduct several numerical experiments. First, I analyze how the enforcement parameter ϕ affects stationary equilibrium variables relative to the full enforcement benchmark. Second, I study the effects of an increase in the output subsidy τ to producers of the composite tradeable good on stationary equilibria in economies with different values of ϕ. To assess the welfare impact of this kind of policy properly and show how growth occurs along the transition path one needs to conduct an analysis of the transition path between stationary equilibria. I have not yet performed this analysis but it is a priority in future work.

Throughout the quantitative analysis I hold fixed all parameters other than ϕ. The purpose of this is to isolate the effects of financial development and its interaction with undervaluation, controlling for all other variables that differ across economies. The first exercise establishes a link between economic development and financial development in the model: an increase in the enforcement parameter ϕ causes all aggregate variables to fall. Output in the domestic tradeable sector falls more than output in the nontradeable sector because an inccrease in ϕ causes larger changes in firm life cycle dynamics in the tradeable sector. The second exercise studies the impact of a subsidy to producers of the composite tradeable good on stationary equilibria. A subsidy causes GDP and output in the tradeable sector to rise, with larger increases in economies with larger values of ϕ. Consumption falls for low values of ϕ but rises for higher values. However, the quantitative impact of the subsidy is small, especially the impact on consumption. This suggests that the welfare impact of the policy is likely to be negligible, although as mentioned above a transition analysis must be performed to make a definitive statement on welfare.

4.1 Calibration

I calibrate the model's non-enforcement parameters so that the stationary equilibrium in the perfect-enforcement verison of the model matches key moments of U.S. macroeconomic data. The underlying assumption is that that the U.S. has a well-functioning financial system. While recent history indicates that this assumption may not be entirely well-founded, there are two reasons that this calibration strategy is sensible. First, because the interaction between financial development undervaluation is the focus of this paper we want to isolate the effects of the enforcement parameter ϕ on the model's quantitative response to an increase in the subsidy τ. This makes it desirable to calibrate the model's non-enforcement parameters indepdendently of ϕ. This is similar to the approach taken by Buera et al. (2009), who study the extent to which financial frictions can account for cross-country TFP differences. Second, this is the same assumption made by Rajan and Zingales (1998) in constructing their measure of external dependence. Since this measure

is the target for the external dependence parameters κ_j it makes sense to use a model environment that makes a similar assumption.

Standard values for several parameters are taken from the literature. The interest rate r^* is set to 4% (annual) and the discount factor β of the households and lenders is set to $\frac{1}{1+r^*}$. The depreciation rate δ is set to 0.06. The survival parameter ψ is set to 0.9 to reflect the exit rate of 10% reported by the U.S. Census Bureau in the Business Dynamics Statistics. The returns to scale $\alpha + \theta$ is set to 0.9 which is taken from Basu and Fernald (1997). The capital share α is set to 0.34. The elasticities of substitution $\frac{1}{1-\rho}$ and $\frac{1}{1-\eta}$ are taken from Kehoe and Ruhl (2009) as 0.5 and 1.5 respectively. The scale parameters z^T and D are chosen so that in equilibrium the price of domestic intermediates and imported intermediates are equalized: $p_N = p_D = p_M = 1$. This is simply a normalization for the units of each of these goods. This leaves the share parameters (a_N, a_T) and (b_D, b_M) and the external dependence parameters κ_j. The calibration targets for these parameters are the ratio of services value added to GDP, imports to GDP, the external dependence values for services and manufacturing calculated by Buera et al. (2009) and the ratio of young firm's external dependence to that of mature firms reported by Rajan and Zingales (1998). Table 4 summarizes the calibration.

Table 4: Baseline parameter values

Parameter	Value	Target	Value
r^*, β	0.9615	U.S. real interest rate	4.0%
δ	0.06	U.S. depreciation to GDP	10%
$\frac{1}{1-\rho}$	0.5	Traded-nontraded elasticity of substitution	0.5
a_N	0.33	Services value added to GDP	45.7%
z_T	1.53	Normalization: $p_N = p_D$	
b_M	0.25	Imports to GDP	15.3%
$\frac{1}{1-\eta}$	1.5	Domestic-import elasticity of substitution	1.5
D	0.24	Normalization: $p_D = p_M$	
ψ	0.9	Exit rate	10%
$\alpha + \theta$	0.9	Basu and Fernald (1997)	0.9
$\frac{\alpha}{\alpha+\theta}$	0.34	Capital share of	0.34%
(e_N, κ_N)	$(0.04, 0.0005)$	External dependence, services; young/mature ex. dep. ratio	0.9; 10
(e_D, κ_D)	$(0.1, 0.004)$	External dependence, manufacturing; young/mature ex. dep. ratio	0.21; 10

## 4.2 Stationary equilibrum comparison

4.2.1 Varying the enforcement parameter

To assess the degree to which the model accounts for the relationship between financial and economic development, I solve for stationary equilibria at different values of ϕ and compare how economic aggregates compare to the full enforcement case. Figure 6 shows GDP per capita, consumption per capita and output in the intermediate sectors for $\phi \in [5, 10]$[5]. We can see that increasing ϕ causes all economic aggregates to decrease. Over this range for ϕ, GDP per capita ranges from 75% to 78% of the perfect enforcement outcome while consumption ranges from 88% to 90.5%. Looking at sector-level output, nontradeable output ranges from 75.5% to 78% of the perfect enforcement outcome while domestic tradeable output ranges from 71% to 74%. So we can see that increasing the enforcement problem affects the tradeable sector more than the nontradeable sector, just as the partial equilibrium analysis suggests. In sum, the limited enforcement problem in the model has large effects on aggregate outcomes, although the magnitude of this effect does not vary widely as ϕ changes.

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[5]For low but positive values of ϕ the enforcement constraint does not bind because the lender's value function is sufficiently high that it is profitable for lenders to pay the entry cost and offer firms an initial equity of v_j^*. There are two points of discontinuity: the first where the enforcment constraint begins to bind in the tradeable sector and the second when the constraint in the nontradeable sector begins to bind. For my purposes I want to study only the region where the enforcement constraints bind in both sectors before and after any policy changes. I pick a lower bound of ϕ that is high enough to guarantee this.

Figure 6: Aggregate impact of the enforcement parameter ϕ
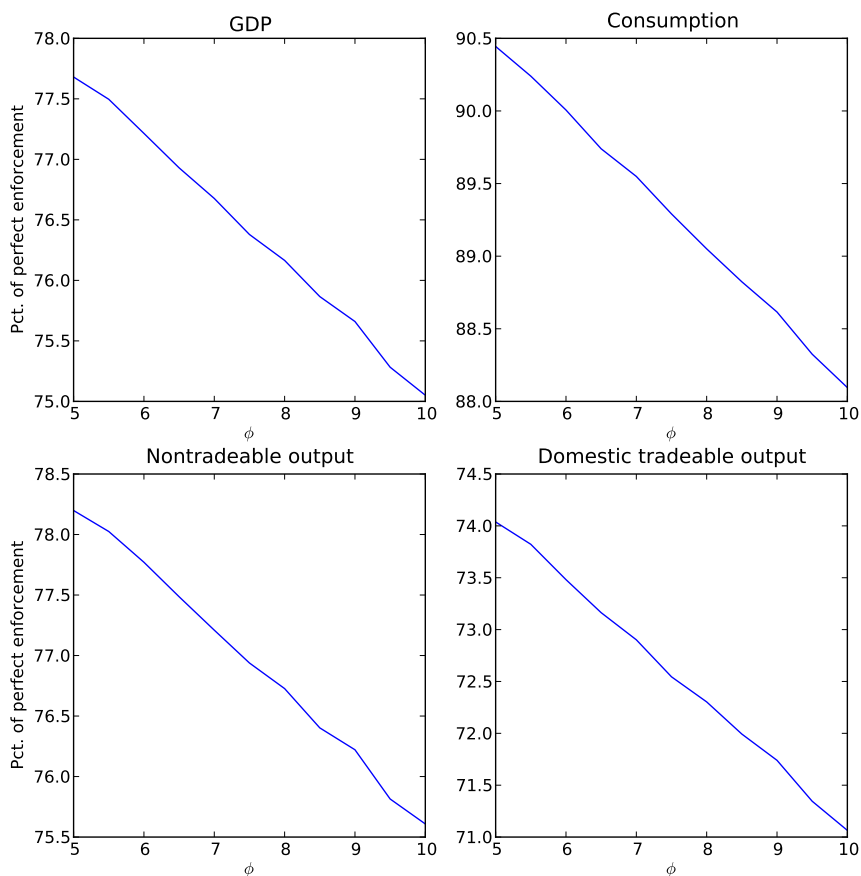

To better understand the aggregate quantitative effects of changing ϕ I look at firm dynamics. Figure 7 plots the effect of increasing ϕ on the equity level sufficient to operate efficiently v^*, the number of periods to reach v^* and the probability that a new entrant will survive to reach v^*. We can see that v^* is increasing in ϕ for both sectors, but the effect is larger for tradeable firms. As ϕ ranges from 5 to 10, v_N^* goes from 0.5 to 4, while v_D^* goes from 3 to 9. The number of periods it takes to reach v^* changes dramatically as we vary ϕ from 5 to 10. In the nontradeable sector this number goes from 9 to 15, while it goes from 11 to 18 in the tradeable sector. This implies into large changes in the probability that firms survive to operate efficiently. In the nontradeable sector this survival probability falls from 43% to less than 25%, while in the tradeable sector it falls from 35% to 17%.

Figure 7: Impact of ϕ on firm life cycle
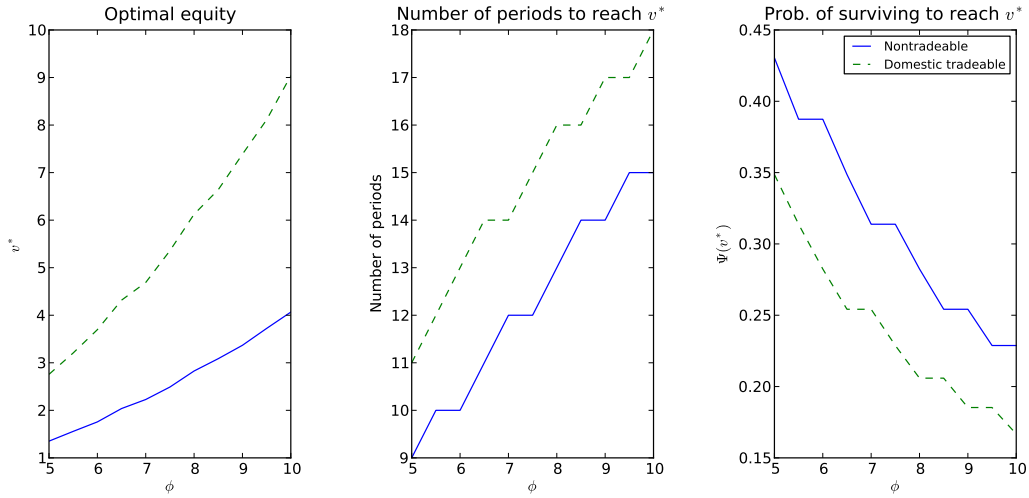
Goods and factor prices change significantly in general equilibrium from those that prevail in the full enforcement version of the model as ϕ increases. The prices of both domestically produced intermediate goods relative to the imported good rise significantly, but the relative price of domestic tradeables rises more. Figure 8 shows how the price of domestic tradeables relative to the price of nontradeables varies with the enforcement parameter. We can see that as ϕ rises domestic tradeables become more expensive relative to nontradeables. This pattern fits with the empirical observation of Buera et al. (2009) that the price of manufactured goods relative to the price of services is negatively correlated with GDP per capita. In the model, this pattern occurs because the supply of domestic tradeables relative to the supply of nontradeables is decreasing in ϕ which causes the marginal rate of substitution between the two (and thus the ratio of prices) to rise.

Figure 8: Impact of ϕ on relative prices

Price of domestic tradeable relative to domestic nonrtadeable
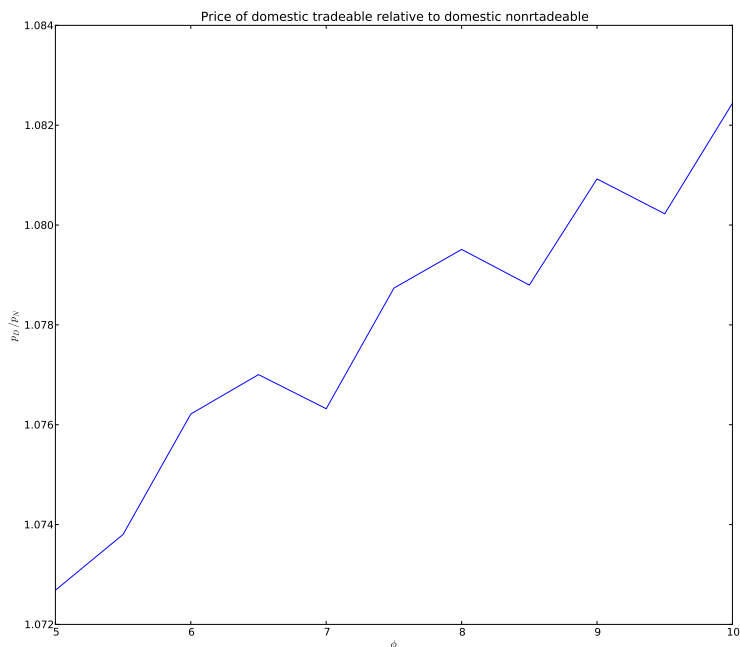
Due to price changes the efficient scale rises dramatically in both sectors. Figure 9 illustrates this point. While firms in both sectors produce much less than the full enforcement output when they enter the market, they produce much more by the time they reach maturity. In other words, while the average firm size falls in both sectors as ϕ increases the largest firms (those that are unconstrained) actually grow significantly. Because the price of domestic tradeables rises relative to the price of nontradeables the magnitude of these changes in scale is larger in the tradeable sector.

Figure 9: Impact of ϕ on firm scale
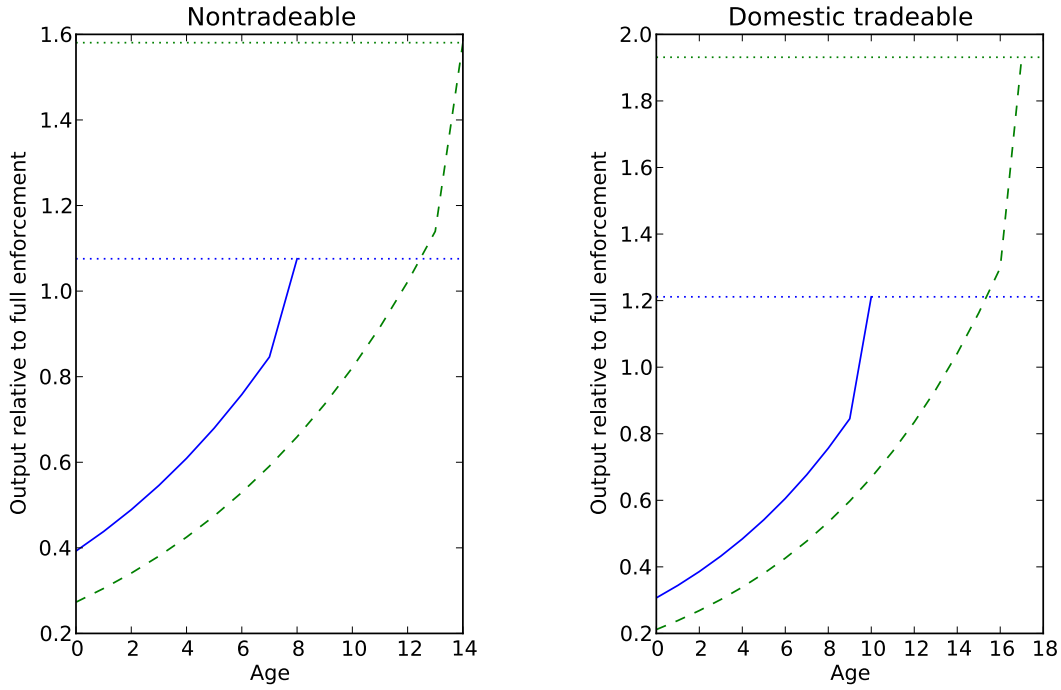
### 4.2.2 Effects of composite tradeable subsidies

To assess the impact of real exchange rate undervaluation induced by government policy on equilibrium outcomes in the model I introduce a 10% subsidy to producers of the composite tradeable good financed by lump-sum taxes on households. Figure 10 shows how the percent changes in aggregate variables vary with the enforcement parameter ϕ. The subsidy raises the real exchange rate by between 7.5% and 8%. We can see that GDP and output in the domestic tradeable sector rise for all values of ϕ, with larger increases for higher values of ϕ. Output in the nontradeable sector falls across the board but falls less for higher values of ϕ. While consumption falls for lower values of ϕ it rises for higher values. This is because the taxes to fund the subsidy wash out any gains from increased output for lower values of ϕ, while for higher values of ϕ output rises enough to grant a small increase to consumption. However, the change in consumption is small for all values of ϕ indicating that the welfare effects of the policy are likely to be negligible.

Figure 10: Aggregate impact of subsidy
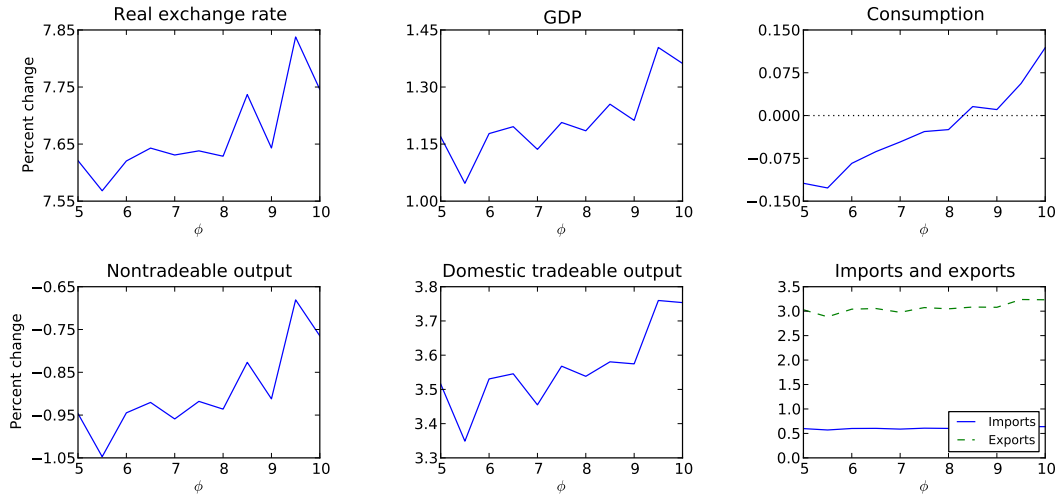
To confirm that the policy intervention alleviates the symptoms of the enforcement problem I examine the impact of the policy on firm life cycle dynamics. Figure 11 shows the effects of the policy on the level of equity required to operate at the unconstrained optimal level, the number of periods before enforcement constraints stop binding and the probability that firms will survive long enough to reach that point. We can see that v_j^* and the number of periods required to reach it fall in both sectors while the probability of reaching maturity rises. The changes to the number of periods to reach v^* and the probability of reaching it are larger in the tradeable sector, although no clear pattern emerges between the size of these changes and the enforcement parameter ϕ.

Figure 11: Impact of subsidy on firm life cycle dynamics
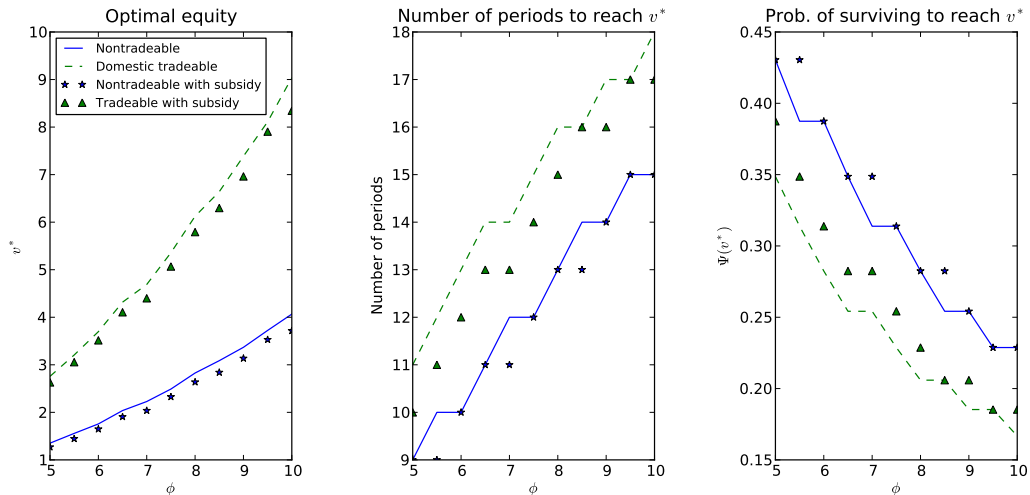
5   Concluding remarks and directions for future research

This paper studies the hypothesis that policy-induced real exchange rate undervaluation can alleviate the effects of limited enforcement of financial contracts, acting as a temporary substitute for institutional reform. This hypothesis is motivated by a recent empirical finding by Rodrik (2008) that undervaluation is linked to increased growth in real GDP per capita in developing countries. To provide further motivation I demonstrate a link between undervaluation and growth in the average external dependence of manufacturing, as well as a link between undervaluation and increased growth in individual manufacturing sectors that rely heavily on external dependence in the sense of Rajan and Zingales (1998). Because frictions associated with poor financial development constraint externally dependent sectors more, this suggests that the channel through which undervaluation promotes growth is related to financial development.

To evaluate the hypothesis of the paper I present a model of a semi-small open economy with multiple sectors, firm entry and exit, and limited enforcement of financial contracts. A key assumption of the model, backed by empirical findings of Buera et al. (2009), is that the tradeable sector is more externally dependent than the nontradeable sector. The limited enforcement problem affects life cycle dynamics of firms, reducing the size at which they enter the market and lengthening the time it takes for them to reach maturity, reducing the probability that firms will survive long enough to become unconstrained. These effects are larger in the tradeable sector because it is more externally dependent. I find that a government policy of subsidizing producers of the composite tradeble good (equivalently, subsidizing the purchase of domestic and foreign tradeable goods) generates undervaluation and reduces the effects of the financial friction, raising GDP per capita. The effects of the policy are larger in economies with more severe enforcement problems, indicating that the hypothesis is sound in a qualitative sense. However, the quantitative impact of the policy in the calibrated model is small, especially for consumption. Moreover, the policy only increases consumption for economies with severe enforcement problems, while consumption falls following the introduction of the subsidy in economies with milder enforcement problems. Overall, the quantitative exercise indicates that undervaluation policy is not an important priority in developing countries.

One primary reason that the model performs poorly in a quantitative sense is that firms only differ in age; there is no heterogeneity in productivity. This means that the model says nothing about the impact of financial frictions and undervaluation policy on the number of potential firms that choose to enter the market. Including this kind of heterogeneity would therefore allow the model to generate additional extensive margin effects that would likely improve its performance. This is the main area of improvement for this study going forward.

In thinking about the welfare impact of financial frictions in the model and undervaluation policy that

alleviates them it is important to keep in mind that the representative agent framework I employ abstracts from differential welfare effects across sectors and economic classes. In other words, the model says nothing about differences in welfare impact across agents employed in different sectors of the economy. Because both the financial friction and the policy in question have different effects on the model's sectors the welfare impacts will not be evenly distributed. One might suspect that workers and especially entrepreneurs working in the tradeable sector would benefit from the subsidy in the model at the expense of workers in the nontradeable sector. Adding heterogeneity in this dimension is another direction in which this study could go.

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A Proofs

Proof of Lemma 1. Fix v and let $T_j(v; \phi)$ denote the number of periods it takes to reach $v_j^*(\phi)$ from v in an economy with enforcement parameter value ϕ (for brevity I will refer to such an economy as a ϕ-economy). We can define it as follows:

$$T_j(v; \phi) = \min \left\{ t : \ v \geq \left(\frac{\psi}{1 + r^*} \right)^t v_j^*(\phi) \right\}$$

Since $v_j^*(\phi_L) < v_j^*(\phi_H)$ it is clear that $T_j(v; \phi_H) \geq T_j(v; \phi_L)$ (equality can happen if ϕ_H is close enough to ϕ_L since $T_j(v; \phi)$ must be an integer). Now define the intratemporal profit function $\pi_j(v; \phi)$ as

$$\pi_j(v; \phi) = \max_{k,l} \ \{ p_j f(k, \ell) - w\ell - rk - p\kappa_j : \ v \geq \phi(rk + p\kappa_j) \}$$

and let π_j^* denote the unconstrained optimal intratemporal profit. It should be obvious that $\pi_j(v; \phi_H) \leq \pi_j(v; \phi_L)$ for all v with strict inequality for $v < v_j^*(\phi_H)$. We can write the lender's value in a ϕ-economy as

$$B_j(v; \phi) = \underbrace{\left[\sum_{t=0}^{T_j(v;\phi)-1} \left(\frac{\psi}{1 + r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1 + r^*} \right)^{-t} v; \phi \right) \right]}_{\text{constrained profit}} + \underbrace{\left[\left(\frac{\psi}{1 + r^*} \right)^{T_j(v;\phi)} \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) \right]}_{\text{unconstrained profit}} - v$$

If $v \geq v_j^*(\phi_H)$ firms in both sectors operate at the unconstrained optimal level (which is the same in both economies) immediately so we simply have

$$B_j(v; \phi_L) = B_j(v; \phi_H) = \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) - v$$

On the other hand, for $v < v_j^*(\phi_H)$ we can write the lender's value in the ϕ_H economy as

$$B_j(v; \phi_H) = \left[\sum_{t=0}^{T_j(v;\phi_L)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_H \right) \right]$$
$$+ \left[\sum_{t=T_j(v;\phi_L)}^{T_j(v;\phi_H)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_H \right) \right]$$
$$+ \left[\left(\frac{\psi}{1+r^*} \right)^{T_j(v;\phi_H)} \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) \right]$$
$$- v$$

Since $\pi_j(v; \phi_H) < \pi_j(v; \phi_L)$ for all $v < v_j^*(\phi_H)$, it must be the case that

$$\left[\sum_{t=0}^{T_j(v;\phi_L)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_H \right) \right] < \left[\sum_{t=0}^{T_j(v;\phi_L)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_L \right) \right]$$

Moreover, it should be clear that

$$\left[\sum_{t=T_j(v;\phi_L)}^{T_j(v;\phi_H)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_H \right) \right] + \left[\left(\frac{\psi}{1+r^*} \right)^{T_j(v;\phi_H)} \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) \right] \leq \left[\left(\frac{\psi}{1+r^*} \right)^{T_j(v;\phi_L)} \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) \right]$$

with strict inequality if if $T_j(v; \phi_H) > T_j(v; \phi_L)$. Note that if $T_j(v; \phi_H) = T_j(v; \phi_L)$, we can write the difference between $B_j(v; \phi_L)$ and $B_j(v; \phi_H)$ as simply

$$B_j(v; \phi_L) - B_j(v; \phi_H) = \left[\sum_{t=0}^{T_j(v;\phi_L)-1} \left(\frac{\psi}{1+r^*} \right)^t \left(\pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_L \right) - \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; \phi_H \right) \right) \right] > 0$$

This inequality holds because, once again, $\pi_j(v; \phi_L) > \pi_j(v; \phi_H)$ for $v < v_j^*(\phi_H)$. Regardless of which case holds, we have shown that $B_j(v; \phi_H) < B_j(v; \phi_L)$ for $v < v_j^*(\phi_H)$.

$\square$

Proof of Proposition 1. As mentioned in the text, initial equity must be in the region where the lender's value is decreasing; otherwise lenders could profit by offering more initial equity to firms. In this region, there is a unique value $v_j^0(\phi_L)$ that satisfies $B_j(v_j^0(\phi_L); \phi_L) = pe_j$. Lemma 1 implies that $B_j(v^0(\phi_L); \phi_H) < pe$. Since $B_j(v, \phi_H)$ is decreasing in this region, it must therefore be that $v_j^0(\phi_H) < v_j^0(\phi_L)$. $\square$

Proof of Lemma 2. The proof of this lemma is very similar to the proof of lemma 1 so I will cover only the differences in detail. Since $\kappa_D < \kappa_N$ we know that $v_D^* > v_N^*$. Fix v. This implies that $T_D(v) \geq T_N(v)$

(with equality possible if κ_D is close to κ_N. We can use the same intratemporal profit function defined in the previous lemma (with the indexing on ϕ omitted as it is not necessary in this context). Since the output prices are the same in both sectors, the only difference between $\pi_D(v)$ and $\pi_N(v)$ is that $\kappa_D > \kappa_N$ which reduces revenue in the tradeable sector relative to the nontradeable sector. Note that this means $\pi_D^* = \pi_N^* = \pi^*$. It should be clear that $\pi_D(v) \leq \pi_N(v)$ with strict inequality for $v < v_D^*$. We can write out the lender's value in terms of discounted revenues minus promised equity in the same way as before. Again, for $v \leq v_D^*$ it is obvious that $B_D(v) = B_N(v)$ since $\pi_D^* = \pi_N^*$.

For $v < v_D^*$ the method employed in the previous lemma carries over almost completely. We have

$$
\begin{aligned}
B_D(v) = & \left[\sum_{t=0}^{T_N(v)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_D \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v \right) \right] \\
& + \left[\sum_{t=T_N(v)}^{T_D(v)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_D \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v \right) \right] \\
& + \left[\left(\frac{\psi}{1+r^*} \right)^{T_D(v)} \left(\frac{\pi^*}{1 - \frac{\psi}{1+r^*}} \right) \right] \\
& - v
\end{aligned}
$$

The first term is less than $\left[\sum_{t=0}^{T_N(v)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_N \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v \right) \right]$ since $\pi_D(v) < \pi_N(v)$ for all $v < v_D^*$. If $T_D(v) = T_N(v)$ we are done. If not, the sum of the second two terms is less than $\left[\left(\frac{\psi}{1+r^*} \right)^{T_N(v)} \left(\frac{\pi^*}{1 - \frac{\psi}{1+r^*}} \right) \right]$ so $B_D(v) < B_N(v)$ in either case. $\qquad\square$

Proof of Proposition 2. The proof is the same as that of proposition 1 if $e_D = e_N$. If $e_D > e_N$ it is obvious that v_D^0 is pushed even further below v_N^0. $\qquad\square$

Proof of Lemma 3. I refer to the previous proofs for definitions of $\pi_j(v; p)$ and $T_j(v; p)$. They are the same as before, only that I index them by the final good price p. Fix v. First, note that $\Delta \pi_j(v; p) \geq 0$, with strict inequality for $v < v_j^*(p)$. This is because the term $p\kappa_j$ in the enforcement constraint falls as the final good price p falls, allowing the firm to rent more capital at a given level of equity. Moreover, $\Delta \pi_D(v; p) > \Delta \pi_N(v; p)$ for $v < v_D^*(p)$. This is due to two things. One, since $\kappa_D > \kappa_N$ the amount of capital firms can rent at the initial price p is lower in the tradeable sector, so concavity of the production function implies that a fixed change in the amount of capital firms can rent will generate a larger increase in output in the tradeable sector. Two, the change in the amount of capital firms can rent (holding equity fixed) is proportional to the $\Delta p \kappa_j$, the change in the fixed component of the enforcement constraint. Since $\kappa_D > \kappa_N$ this magnitude of this change is larger in the tradeable sector, and so is the change in the amount of capital firms can rent.

These two facts combine to imply that $\Delta \pi_D(v; p) > \Delta \pi_N(v; p)$.

Second, note that $\Delta v_j^*(p) < 0$, i.e., the level of equity required to produce efficiently falls. This is due to the fact that the fixed component in the enforcement constraint $p\kappa_j$ falls. This means that the number of periods it takes to reach $v_j^*(p)$ from v weakly falls (it can stay constant if the size of the change in the fixed component $\Delta p\kappa_j$ is small). Moreover, $\Delta v_D^* < \Delta v_N^*$, i.e., the level of equity required to produce efficiently falls more in the tradeable sector since $\Delta p\kappa_j$ falls more in the tradeable sector. This implies that the number of periods it takes to reach v_j^* from v falls weakly more in the tradeable sector: $\Delta T_D(v; p) \leq \Delta T_N(v; p)$ (again, it can happen that this change is the same in both sectors if κ_D is close to κ_N).

Recall from the proof of lemma 1 that we can write $B_j(v; p)$ as

$$B_j(v;p) = \left[\sum_{t=0}^{T_j(v;p)-1} \left(\frac{\psi}{1+r^*} \right)^t \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; p \right) \right] + \left[\left(\frac{\psi}{1+r^*} \right)^{T_j(v;p)} \left(\frac{\pi_j^*}{1 - \frac{\psi}{1+r^*}} \right) \right] - v$$

We can write the change in the lender's value function $\Delta B_j(v; p)$ as

$$\Delta B_j(v;p) = \left[\sum_{t=0}^{T_j(v;p')-1} \left(\frac{\psi}{1+r^*} \right)^t \Delta \pi_j \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; p \right) \right]$$
$$+ \sum_{t=T_j(v;p')}^{T_j(v;p)} \left[\left(\frac{\psi}{1+r^*} \right)^t \left(\pi^* - \left(\left(\frac{\psi}{1+r^*} \right)^{-t} v; p \right) \right) \right]$$

where $p' = p + \Delta p$. The first term is the discounted change in profits during periods in which the firm is constrained before and after the price change. The second term is the change in profits during periods in which the firm was constrained before the price change but unconstrained afterwards. We know that $\Delta \pi_j(v; p) > 0$ which implies the first term is strictly positive. We also know that $\Delta T_j(v; p)$ is weakly negative which implies that the second term is weakly positive. Summing up, we can see that $\Delta B_j(v; p) > 0$. Moreover, we know that $\Delta \pi_D(v; p) \geq \Delta \pi_N(v; p)$ with strict inequality for $v < v_D^*(p)$. This means that the first term is strictly larger in the tradeable sector. We also know that $\Delta T_D(v; p) < \Delta T_N(v; p)$. This means that the second term is weakly larger in the tradeable sector. Thus $\Delta B_D(v; p) > \Delta B_N(v; p)$. $\qquad \square$

Proof of Proposition 3. From lemma 3 we know that $\Delta B_j(v; p) > 0$. Holding fixed the size of the entry cost pe_j for a moment, this means that inital equity $v_j^0(p)$ rises since $B_j(v; p)$ is decreasing in the region where it intersects the entry cost. In addition, the fact that $\Delta pe_j < 0$ (i.e., the size of the entry cost falls) in addition to the increase in $B_j(v; p)$ means that the equity level at which the new value function $B_j(v; p')$ intersects the entry cost $p'e_j$ rises even more. Therefore $\Delta v_j^0(p) > 0$.

Moreover, we know from 3 that $\Delta B_D(v; p) > \Delta B_N(v; p)$. Further, the size of the drop in the entry cost

is larger in the tradeable sector since $e_D > e_N$, i.e., $\Delta p e_D > \Delta p e_N$. These two facts combine to imply that the change in initial equity is larger in the tradeable sector: $\Delta v_D^0(p) > \Delta v_N^0(p)$. $\qquad\square$