

# Export Market Penetration Dynamics

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## Abstract

The entry, exit, and expansion of exporting firms plays a key role in driving aggregate trade fluctuations. This paper uses Brazilian microdata to show that exporters' life cycles vary significantly across foreign markets: in destinations with higher export participation, overall turnover is lower but new exporters are smaller and exit more frequently. To account for these facts, this paper develops a novel theory of exporter dynamics that combines insights from two approaches: the static quantitative trade literature, which emphasizes the role of geography in shaping exporting costs; and the dynamic trade literature, which emphasizes the role of sunk costs in shaping forward-looking decisions to start and stop exporting. When calibrated to match the data, the model predicts stronger responses to trade reforms, more pronounced exchange-rate hysteresis, and larger consequences of trade policy uncertainty in markets with lower export participation.

## 1 Introduction

Trade dynamics are driven by individual firms' decisions: whether to start or stop exporting; whether to expand to new foreign markets; and whether to expand their operations in existing ones. The static quantitative trade literature emphasizes the influence of economic geography on the distribution of exporting firms and long-run consequences of trade reforms. In smaller, poorer markets, there are fewer small exporters and sales are less concentrated among the largest ones, and trade is more sensitive to changes in trade costs (Arkolakis, 2010; Eaton et al., 2011; Kehoe and Ruhl, 2013). The trade dynamics literature highlights how turnover amongst exporters and differences in performance between new exporters and incumbents shapes transition dynamics. Export participation is highly persistent and new exporters are smaller and less likely to survive, and trade adjusts slowly over time in response to trade reforms and other shocks as a result. (Alessandria and Choi, 2007; Ruhl and Willis, 2017; Alessandria et al., 2014). In this paper, I ask: how do the microeconomic dynamics of exporting firms vary across the markets to which they sell? What economic forces drive these differences? What are the implications for aggregate trade dynamics?

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In the empirical part of the paper, I use customs data from Brazil to document new facts about exporters' life cycles and how these facts vary across export destinations. The cross-sectional distribution and life-cycle dynamics of Brazilian exporters are consistent with facts that have been reported elsewhere in the literature using different data sources. Brazilian exports are highly concentrated—large firms that serve many destinations account for the bulk of export volumes—and new entrants to an export market sell less than incumbents, are more likely to exit, and grow more rapidly. I use these data to deliver new insights about how exporters' life cycles depend on the characteristics of the destinations that they serve. In “easier” destinations—large, rich markets like the United States—turnover is lower and new exporters are smaller and exit more frequently relative to incumbents than in “harder” destinations—smaller, poorer countries like Vietnam. Successful exporters that survive for many years in a market before exiting sell more upon entry and experience more sales growth over their tenures than less successful exporters that survive for only a few years, and these differences, too, are more pronounced in easier destinations than in harder ones. There is also substantial variation in export performance within individual firms' “portfolios” of export destinations.. Multi-destination exporters—exporters that serve more than one destination—are more likely to exit from the destinations to which they sell the least, but exporters that serve 10 or more destinations are less likely to exit even from their least important markets than single-destination exporters.

In the theoretical part of the paper, I develop a novel theory of export participation dynamics that can account for the full range of stylized facts about the cross-sectional distribution and life-cycle dynamics of exporters. The model extends the endogenous market penetration framework of Arkolakis (2010) to a dynamic environment. To export to a foreign market a firm must advertise, and more intensive advertising efforts reach more foreign customers. Two key properties allow the model to deliver life-cycle dynamics that are consistent with the data. First, the marginal cost of reaching additional customers in the future is decreasing in a firm's current customer base. This property implies that firms choose endogenously to enter a market with a small customer base and grow gradually over time. Second, firms with smaller customer bases are more likely to exit endogenously. Coupled with the first property, this implies that a firm's exit rate is decreasing with its tenure in a given export destination. As in the static Arkolakis (2010) model, there are increasing returns to market size in advertising, which means that firms achieve higher market penetration rates in larger markets. This implies that a multi-destination firm will exit more often from small destinations in which it has the fewest customers and exports the least. Despite its parsimony, the model is able to account for all of the key patterns described above.

In the quantitative part of the paper, I calibrate the model so that it reproduces the facts the facts described above. I simulate a panel of firms for each destination, changing only the destinations' populations, income per capita, and trade costs, and choose values for the model's parameters so that the moments computed using the simulated data match the actual moments observed in the customs data. I then use the calibrated model to perform hypothetical trade reform exercises to investigate the implications of the

theory for aggregate trade adjustment dynamics. For each of the destinations in my analysis, I simulate the transition dynamics that follow permanent trade reforms, temporary exchange rate shocks, and trade policy uncertainty episodes. I find that in harder destinations, trade flows respond more in the long run but take longer to adjust; exhibit more pronounced hysteresis following temporary exchange rate depreciations; and are affected more by trade policy uncertainty. To corroborate these findings, I use the customs data to analyze how Brazilian bilateral trade flows responded to the large real exchange rate depreciation during 1999–2003. Consistent with the model’s predictions, I find that trade grew more—and more slowly—following this depreciation in destinations with lower export participation after controlling for changes in destinations’ price levels, incomes, and multilateral imports.

This paper makes several contributions to the international trade literature. A number of recent studies, such as Ottaviano and Mayer (2007), Eaton et al. (2011), Bernard et al. (2012), have documented that export participation rates and the cross-sectional distribution of trade volumes across exporters vary systematically with the characteristics of export destinations. To account for these patterns, Arkolakis (2010) develops a theory of endogenous market penetration in which firms advertise to reach foreign customers. The marginal cost to reach additional customers is increasing, which implies that low-productivity firms reach fewer customers in a given export market than high-productivity firms, and advertising is more effective in larger markets, which implies that larger markets have higher export participation rates and more small exporters. In this paper, I draw on these insights to develop a theory of market penetration dynamics that accounts for stylized facts about the life-cycle dynamics of exporters as well as facts about the cross-sectional distribution.

Other studies have documented that new entrants to an export market sell less than incumbents, are more likely to exit, and grow more rapidly (Bernard and Jensen, 2004; Eaton et al., 2007a; Alessandria et al., 2014; Ruhl and Willis, 2017; Fitzgerald et al., 2016). In this paper, I use microdata on Brazilian exporters to document that these patterns, too, vary systematically with the characteristics of export destinations. Without conditioning on tenure, exporters exit less frequently from larger export markets than from smaller ones. However, new entrants to larger markets exit more frequently and sell less compared to incumbents than entrants to smaller markets. Moreover, “many-destination” exporters that serve 10 or more destinations are less likely to exit, even from their least important destinations, than exporters that serve one only destination. My theory of market penetration dynamics includes two key features that allow it to account for these facts. First, the marginal cost of reaching additional customers in an export market is decreasing in both the size of that market and the firm’s current customer base. Second, the probability that an exporter exits a market is decreasing in its customer base. These properties imply that the larger markets attract less productive exporters, who reach only a few foreign customers on entry and thus exit frequently, but surviving entrants in these markets grow rapidly. Conversely, smaller markets attract only the most productive exporters, who can afford to reach many customers on entry and thus exit less often.

Sunk-cost models in which heterogeneous firms face large costs of entering an export market and small

costs to continue exporting are often used to study the life cycle dynamics of exports and analyze the macroeconomic implications of these dynamics (Das et al., 2007; Alessandria and Choi, 2007; Alessandria et al., 2014; Ruhl and Willis, 2017). These models, however, cannot account for the gradual growth in sales that occurs over an exporter's tenure in a market or the fact that entrants are more likely to exit, except for variants like Ruhl and Willis (2017) and Alessandria et al. (2014) in which demand grows exogenously with time in a market. My theory of market penetration dynamics generates this growth as an endogenous outcome, and also accounts for variation in this growth across firms as well as across markets. The most similar papers in terms of modeling approach are Fitzgerald et al. (2016) and Piveteau (2020), both of which feature endogenous customer accumulation. The advantages of my approach are as follows. First, neither paper explains why entrants are smaller than incumbents. In both papers, all entrants start exogenously with the same number of customers regardless of productivity or demand for their products. Second, both papers require sunk entry costs and fixed continuation costs on top of customer accumulation costs to generate entry and exit, and require substantial exogenous variation in these costs across firms to match the data. In my model, extensive-margin decisions are driven solely by the marginal cost of serving the first customer in a market, which varies endogenously across firms and over time. Third, the parsimony of my approach makes it more amenable to quantitative analysis. In fact, it is tractable even in general equilibrium; Steinberg (2019) uses an early version of the model in a multi-country DSGE model to study the consequences of uncertainty about Brexit. Most importantly, though, my approach accounts for variation in exporter performance dynamics across destinations—Fitzgerald et al. (2016) and Piveteau (2020) do not explore or attempt to explain this variation at all.

Microdata-disciplined models are invaluable tools to understand fluctuations in trade flows and evaluate the implications of trade policy changes. For example, the seminal work of Melitz (2003), for example, highlights how trade reallocates resources between firms, and, through this channel, affects aggregate productivity. For another, sunk-cost models of export participation dynamics explain why trade flows respond less to price changes in the short run than in the long run (Ruhl, 2008; Alessandria and Choi, 2016) and suggest that dynamic gains from trade policy reforms may differ substantially from the long-run gains (Alessandria et al., 2014). For yet another, the market penetration theory of Arkolakis (2010) upon which this paper builds explains why the least-traded products respond most to changes in trade policy (Kehoe and Ruhl, 2013; Kehoe et al., 2015). My theory of market penetration dynamics provides new insights about how export adjustment dynamics depend on export market characteristics. I find that in smaller, poorer destinations, trade is more elastic in the long run but the transition to the long run takes longer. I also find that trade flows take longer to adjust to a reduction in trade costs than an increase.

## 2 Data

In my empirical analysis, I use Brazilian panel data to study how the distribution of exporting firms and the dynamics of these firms' performance over their tenures as exporters vary across destinations. The data source is a record of all Brazilian export transactions from 1996 to 2008. For each transaction, the dataset includes the destination country, the value of the shipment in U.S. dollars, the year and month of the transaction, an 8-digit product code, and a unique firm identifier. I restrict attention to manufacturing industries and destinations that are served by at least 20 firms in a each year following Fernandes et al. (2016).

First, I document the extent to which the cross-sectional distribution of exporters, the exit rate, and the performance of new exporters relative to incumbents vary across destinations, and show that this variation is systematically related to destinations' characteristics. Second, I study how exporters' sales and survival rates grow after they enter a new destination, and how this growth varies across destinations. Third, I study differences in performance within exporters' individual portfolios of destinations.

I have also analyzed similar data on Mexican and Peruvian exporters from the World Bank's Exporter Dynamics Database (Fernandes et al., 2016). These datasets are of somewhat lower quality than the Brazilian data as they contain fewer firms and cover shorter time periods. Nevertheless, as appendix A shows, all of the results documented in this section about Brazilian exporters also apply to both Mexican and Peruvian exporters. This indicates that these results are robust to variation in conditions in the exporting country.

### 2.1 Variation in exporter performance across destinations

I analyze three aspects of the cross-sectional distribution of exporters: the number of firms that export to a destination; the share of sales accounted for by the top 5% of exporters in that destination; and the average number of other destinations to which exporters sell. I also analyze three aspects' of exporter dynamics: the exit rate; the average size of entrants relative to incumbents; and the exit rate of entrants relative to incumbents. In the spirit of Ruhl and Willis (2017), I use the term "new exporter dynamics" to jointly refer to the differences in entrants' sizes and survival rates relative to incumbents.

Panel (a) of table 1 reports summary statistics for each of these measures. The first three columns show that the cross section of exporting firms varies widely across destinations. The most popular export destination, Argentina, has an export participation rate more than 100 times greater than the least popular destination, Vietnam. The top-5 share ranges from 0.27 to 0.59, indicating that exports are highly concentrated among the largest exporters in some destinations and evenly distributed on others. The average number of other destinations exporters serve varies from less than 8 to almost 30, suggesting that some destinations are served only by firms with a large portfolio of other destinations. The last three columns

of the table show that exporter dynamics also vary across destinations. Exit rates range from as low as 30 percent to as high as 60 percent, and while entrants are smaller and more likely to exit than incumbents in all destinations, these differences are more pronounced in some destinations and more muted in others.

Figure 1 shows that this variation is systematic: in destinations with higher export participation rates, sales are more concentrated among top exporters, the average exporter sells to fewer other destinations, exporters are less likely to exit, and entrants are smaller and more likely to exit relative to incumbents. The association between export participation and sales concentration is consistent with earlier findings documented by Arkolakis (2010) and Eaton et al. (2011) using French data,<sup>1</sup> and the relationship with the exit rate is consistent with evidence from Mexico reported by Mix (2020a). To my knowledge, however, the fact that new exporter dynamics vary systematically with export participation is new to the literature.

The export participation rate is a convenient statistic that summarizes the popularity of a given destination among exporting firms, but like the other measures shown in table 1 it is ultimately driven by a destination’s exogenous characteristics—both axes in the plots in figure 1 are endogenous outcomes. To dig deeper into the source of the variation in the distribution and dynamics of exporters across destinations, I estimate regressions of the form,

$$M_{j,t} = \alpha + \beta \log L_{j,t} + \log Y_{j,t} + \log \tau_{j,t} + f_t + \epsilon_{j,t}. \quad (1)$$

The dependent variable,  $M_{j,t}$ , is a measure of exporter performance in destination  $j$  in year  $t$  (e.g. the top-5 share or the exit rate). The independent variables are the destination’s population,  $L_{j,t}$ , and its income per capita,  $Y_{j,t}$ ; a trade barrier,  $\tau_{j,t}$ , that is measured as the residual from a standard gravity regression; and a year fixed effect,  $f_t$ , that controls for multilateral trends (e.g. business cycles, nominal exchange rate depreciation).<sup>2</sup> Panel (a) of table 2 reports the results from these estimations. The first column shows that export participation is increasing in population and income per capita and decreasing in trade barriers. This is not terribly surprising, but it provides support for using export participation rate as a convenient, one-dimensional summary of a destination’s popularity or difficulty. The remaining columns show that the other measures of exporter performance all have the associations with destination characteristics one would expect based on figure 1. In “easy” destinations with large, rich populations and low trade barriers, exports are more concentrated, the average exporter serves only a few other destinations, turnover is lower, and new exporter dynamics are more pronounced. By contrast, in “harder” destinations with small, poor populations and high trade barriers, exports are less concentrated, the average exporter serves many other

<sup>1</sup>These studies also show that export intensity—the ratio of a firm’s sales in a foreign destination to its overall sales—is also increasing with export participation. My dataset does not include information about Brazilian exporters’ domestic sales, so I am unable to conduct a similar analysis.

<sup>2</sup>I have also estimated specifications in which tariffs, distance, and other gravity variables are included directly as independent variables rather than indirectly through a gravity residual. Additionally, I have estimated the relationships between destination characteristics and exporter performance at the industry level, including industry effects to control for variation in the composition of exports across destinations. The results from these specifications are in line with the results reported in this paper, and are available upon request.

destinations, turnover is higher, and new exporters are similar to incumbents.

## 2.2 Variation in exporter performance with time in a market

I analyze how exporters grow over time after they enter new markets following the approach of Fitzgerald et al. (2016) and Fitzgerald and Priolo (2018). I group firms by the number of consecutive years that they export to a particular destination before exiting—the duration of an export “spell”—and then estimate the sales trajectories of firms in each group. Formally, I estimate the following regression:

$$\log ex_{i,j,t} = \alpha + \sum_{m=1}^6 \sum_{n=1}^m \beta_{m,n} \mathbb{1}_{\{\text{duration}_{i,j}=m\}} \mathbb{1}_{\{\text{years in market}_{i,j,t}=n\}} + f_j + f_t + \varepsilon_{i,j,t}, \quad (2)$$

where  $ex_{i,j,t}$  is firm  $i$ 's exports in destination  $j$  in year  $t$ ,  $\text{duration}_{i,j}$  indicates the duration of the firm's export spell in that destination, and  $\text{years in market}_{i,j,t}$  indicates the number of years the firm has consecutively exported so far in that destination. I top-code duration at 6 years (the shortest observation window for a destination in my dataset) and include destination and year fixed effects. The reference group is the set of firms with durations of 1, i.e., firms that export for one year and then exit immediately; the coefficient  $\beta_{m,n}$  captures how much more a firm with duration  $m$  sells after exporting for  $n$  periods than a firm in this group. To study how these export trajectories differ across destinations, I split the data into two subsamples: “hard” destinations below the 50th percentile of export participation; and “easy” destinations above the 50th percentile. I then estimate the above equation for each subsample separately.

Figure 2 shows the results of these estimations, with the results for hard destinations shown in panel (a) and the results for easy destinations in panel (b). Overall, the results mirror those of Fitzgerald et al. (2016). Sales tend to grow with the length of an exporter's tenure in a market, but this growth is strongest for the most successful exporters (those that export for six or more consecutive years before exiting). There is substantial variation in sales upon entry: exporters with longer spells sell more in their first year in a market than exporters with shorter spells. Finally, sales tend to fall in the year before a firm exits; falling sales indicate that a firm is likely to exit in the near future. Comparing the two panels, we see that all of these findings are more pronounced in easy destinations than in hard ones: there is less growth in sales over the duration of an export spell in the former than in the latter than the former, and the sales of new entrants conditional on spell duration are more compressed. In easy destinations, the most successful exporters' more than double over the first 6 years of their export spells, and when they enter their sales are more than 150 log points greater than those of firms that last only 1 year. In hard destinations, on the other hand, the most successful exporters' sales grow by less than 50 log points over the duration of their spells, and their sales upon entry are about 120 log points greater than one-year exporters' sales. In short, these results confirm that new-exporter dynamics are more pronounced in destinations with greater export participation.

I analyze how the likelihood of continuing as an exporter depends on time in a market using a linear

probability model of the form,

$$\mathbb{1}_{\{\text{exit}_{i,j,t}=1\}} = \alpha + \sum_{n=1}^6 \beta_n \mathbb{1}_{\{\text{years in market}_{i,j,t}=n\}} + f_t + f_j + \epsilon_{i,j,t}, \quad (3)$$

as in Ruhl and Willis (2017). Here, the reference group is new entrants, so the coefficient  $\beta_n$  indicates how much more likely an exporter that has survived for  $n$  years is to exit than a firm that has just begun to export. As before, I split the data into subsamples of hard and easy destinations based on export participation. The results, which are shown in figure 3, confirm findings reported elsewhere in the literature: exit becomes less likely as time in a market grows. Here, though, there is little difference between hard and easy destinations. In both groups, firms that have survived for 6 years are about 40 percentage points less likely to exit than new entrants. Note that this does not imply that firms are equally likely to exit after 6 years in hard and easy destinations, but rather that they are equally less likely to exit than new entrants after controlling for variation across destinations in entrants' exit rates. As figure 1 shows, the overall exit rate is substantially higher in hard destinations than easy ones, and although the difference between entrants' and incumbents' exit rates is smaller in these destinations, on the whole entrants exit more frequently in these destinations than in easy ones.

### 2.3 Variation in performance within exporters' destination portfolios

To analyze how individual exporters' performance varies across the destinations to which they sell, I first group firms by the number of destinations in their export "portfolios." Figure 4 shows the distribution of exporters and their total sales across all destinations by the sizes of their portfolios. 40 percent of exporters only sell to one destination. Of the remaining 60 percent, most sell to between 2–4 destinations. Only 12 percent of firms export to 10 or more destinations, but these firms account for about 75 percent of total exports in any given year, whereas firms that export to 4 or fewer destinations account for barely 10 percent. This finding is consistent with the "superstar" phenomenon documented elsewhere in the literature (Bernard et al., 2012; Eaton et al., 2007b, 2011; Ottaviano and Mayer, 2007).

I then rank the destinations within each firm's portfolio by sales and analyze how firms perform in high- vs. low-ranked destinations. Harder destinations have smaller populations with lower purchasing power and higher trade barriers, and so one would expect these destinations to be ranked lower on average in exporters' portfolios as well as having lower export participation overall. Table 3, which reports associations between destinations' characteristics and their average ranks within exporters' portfolios, confirms that this is the case. The average rank of a destination within an exporter's portfolio is decreasing in population and income per capita, and increasing in trade barriers: firms export less to harder destinations than easy ones.<sup>3</sup> Since harder destinations have higher overall exit rates, one would therefore expect that firms are

<sup>3</sup>Estimating a Poisson or negative binomial regression on the raw firm-level data yields similar results.



more likely to exit lower-ranked destinations within their portfolios. Table 4, which lists exporters' average exit rates broken down by portfolio size (vertical axis) and destination rank (horizontal axis), confirms that this is the case as well: exporters that serve several markets have the highest exit rates in their least important destinations. However, many-destination exporters are less likely to exit from their least important destinations than single-destination exporters are from their sole destination.

To analyze how the rank of a destination within a firm's portfolio affect its sales and likelihood of exit relative to other firms that export to that destination, I estimate regressions of the form

$$\log ex_{i,j,t} = \alpha + \sum_{m=1}^{10} \sum_{n=1}^m \beta_{m,n} \mathbb{1}_{\{\text{num. dests.}_{i,j}=m\}} \mathbb{1}_{\{\text{dest. rank}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t} \quad (4)$$

$$\mathbb{1}_{\{\text{exit}_{i,j,t}=1\}} = \alpha + \sum_{m=1}^{10} \sum_{n=1}^m \beta_{m,n} \mathbb{1}_{\{\text{num. dests.}_{i,j}=m\}} \mathbb{1}_{\{\text{dest. rank}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t}. \quad (5)$$

$$(6)$$

Here, I top-code portfolio size and destination rank at 10. The reference group is the set of firms that serve only one destination, so the coefficient  $\beta_{m,n}$  in each regression measures how much more a firm sells (in the first specification) or how likely it is to exit (the second specification) in a given destination relative to a firm for whom this destination is its only market. Figure 5 shows the results. In panel (a), we see that firms that export to at least 2 destinations sell more in their highest-ranked destination than firms that export to that destination only. The larger a firm's portfolio, the greater the difference: firms that sell to 2 destinations sell about twice as much in their highest-ranked market as single-destination exporters; while firms with portfolios of 10 or more destinations sell about 5 times as much. We also see, however, that sales relative to single-destination exporters fall with a destination's rank. In fact, all firms except those with 10 or more destinations in their portfolios sell less in their lowest-ranked destinations than single-destination firms. In panel (b), we see these patterns reversed for exit rates. Multi-destination firms are less likely to exit from their most important destinations than single-destination firms—as much as 45 p.p. less likely for firms with 10 or more destinations. However, the gap shrinks as a destination's rank with an exporter's portfolio rises; firms that sell to 4 or fewer destinations are actually more likely to exit from their least important destinations than single-destination exporters.

### 3 Model

The model economy consists of one exporting country and  $J$  importing countries that are indexed by  $j = 1, \dots, J$ . The exporting country is populated by a continuum of firms that produce differentiated goods using constant-returns-to-scale technologies. Each importing country is populated by a measure  $L_j$  of identical consumers with income per capita  $Y_j$  and constant-elasticity-of-substitution preferences over imported goods. Trade barriers are captured by iceberg transportation costs,  $\tau_j$ , which also vary across export markets.

As in Arkolakis (2010), firms are heterogeneous in their customer bases in each export market, which they can increase endogenously by advertising. The market penetration cost of depends on a firm’s current customer base, leading firms to gradually penetrate foreign markets over time.

As in Arkolakis (2010) and Ruhl and Willis (2017), I assume that importing countries are large relative to the exporting country so that aggregate prices and quantities in importing countries are independent of outcomes in the exporting country. I also assume that export activities are small relative to the total size of the exporting country’s economy so that the wage in the exporting country, which I normalize to one without loss of generality, is independent of export-sector outcomes. Finally, I assume for the moment that all aggregate variables, including trade barriers and other destination characteristics, are constant to economize on notation; this section restricts attention to the model’s stationary equilibrium. In my quantitative analysis, however, I analyze transition dynamics that follow permanent and temporary changes in trade barriers.

### 3.1 Firm characteristics

There is a unit measure<sup>4</sup> of firms in the exporting country that produce differentiated varieties according to constant-returns-to-scale technologies. Firms are heterogeneous in multilateral productivity,  $x \in \mathcal{X}$ ; bilateral demand,  $\mathbf{z} = (z_1, z_2, \dots, z_J) \in \mathcal{Z}^J$ ; and the fraction of consumers in each market to which they can sell,  $\mathbf{m} = (m_1, m_2, \dots, m_J) \in [0, 1]^J$ .

Productivity is common to all markets, and evolves according to a Markov process with transition function  $G(x', x)$ . Demand in each market  $j$  evolves independently according to a Markov process with transition function  $H(z'_j, z_j)$ . A firm’s customer base in each market is chosen endogenously in a manner that I describe below. Each period, a firm has a chance  $1 - \delta(x)$  of dying, which I allow to depend on its productivity to capture the fact that smaller firms shut down more frequently (Alessandria et al., 2014). When a firm dies, it is replaced by a new firm with productivity and demand shocks drawn from their respective ergodic distributions,  $\bar{G}(x)$  and  $\bar{H}(z_j)$ . Newborn firms have zero customers in all export markets.

### 3.2 Export demand, pricing, and profits

Firms compete monopolistically as in Melitz (2003) and Chaney (2008). Market  $j$ ’s demand for a firm’s product depends on the market’s characteristics,  $L_j$  and  $Y_j$ ; the firm’s price in that market,  $p$ ; the firm’s demand shock in that market,  $z$ ; and the fraction of consumers in that market to which the firm can sell,  $m$ . Conditional on purchasing the firm’s product, an individual consumer in market  $j$  has a standard downward-sloping demand function:

$$c_j(z, p) = L_j Y_j (p/z)^{-\theta}, \quad (7)$$

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<sup>4</sup>I abstract from firm creation in this paper. My focus is on exporter performance in bilateral trade relationships, and the interpretation of this assumption is that the prospect of exporting to a single destination, even a large one, is too small to affect firm creation incentives. Studying the relationship between bilateral trade and firm creation is a promising avenue of investigation that could yield important insights about Brexit, the U.S.-China trade war, and other phenomena, but I leave this for future research.

where the parameter  $\theta$  is the elasticity of substitution between varieties.<sup>5</sup> Total demand for the firm's product in market  $j$  depends on the firm's price as well as the number of customers it can serve as in Arkolakis (2010) and Eaton et al. (2011):

$$q_j(z, m, p) = mc_j(z, p). \quad (8)$$

Each market's idiosyncratic demand for firm's product evolves independently over time according to a

Conditional on its productivity,  $x$ , demand,  $z$ , and its customer base,  $m$ , a firm chooses its price in each market  $j$  to maximize profits,

$$\pi_j(x, z, m) = \max_p \left\{ pq_j(z, m, p) - \frac{\tau_j q_j(z, m, p)}{x} \right\}. \quad (9)$$

The optimal price is given by the standard constant-markup solution,

$$p_j(x) = \frac{\theta}{\theta - 1} \frac{\tau_j}{x}. \quad (10)$$

The firm's exports to market  $j$  and associated profits can be written as

$$ex_j(x, z, m) = \left( \frac{\theta}{\theta - 1} \right)^{1-\theta} mL_j Y_j \tau_j^{1-\theta} (xz)^{\theta-1} \quad (11)$$

and

$$\pi_j(x, z, m) = \frac{1}{\theta} \left( \frac{\theta}{\theta - 1} \right)^{1-\theta} mL_j Y_j \tau_j^{1-\theta} z^{\theta-1} \equiv \tilde{\theta} mL_j Y_j \tau_j^{1-\theta} (xz)^{\theta-1} \equiv \tilde{\pi}_j (xz)^{\theta-1}, \quad (12)$$

respectively.

### 3.3 Market penetration dynamics

A firm's market penetration in each destination evolves over time as it attracts new customers and loses some of its old ones. A firm with market penetration  $m$  that attracts  $n \in [0, 1 - m]$  new customers and retains  $o \in [0, m]$  old ones will have a market penetration of

$$m' = n + o \quad (13)$$

in the next period. Note that as a firm's market penetration grows, the number of potential new customers from which it can draw,  $1 - m$ , shrinks, while the number of customers it can retain grows. I use the term "potential entrant" to refer to a firm with zero market penetration and the term "incumbent" to refer to a firm with positive market penetration. The terms "entrant" and "new exporter" equivalently refer to a potential entrant that attracts at least one new customer; a new exporter becomes an incumbent in the next period.

<sup>5</sup>The price level in each market is normalized to one;  $Y_j$  can be interpreted as purchasing power-adjusted income per capita.

Customer attraction and retention both depend on the firm's advertising efforts. I use  $s$ —for search—to denote advertising targeted at new customers and  $r$ —for retention—to denote advertising targeted at old customers.<sup>6</sup> Following Arkolakis (2010), the marginal effect of search effort on customer attraction is increasing in the total number of potential new customers,  $(1 - m)L_j$ , and decreasing in the fraction of potential new customers a firm successfully attracts,  $n / (1 - m)$ :

$$n'_j(s) = \psi_n L_j^{-\alpha_n} (1 - m)^{-\beta_n} \left( \frac{1 - m - n_j(s)}{1 - m} \right)^{\gamma_n}. \quad (14)$$

The parameter  $\alpha_n \in [0, 1]$  governs returns to population size in advertising to new customers. The smaller  $\alpha$ , the easier it is to attract new customers in larger countries. Similarly  $\beta_n$  governs the returns to scale with respect to the size of the pool of potential new customers in a particular market. The smaller  $\beta_n$ , the easier it is for a firm to attract new customers when its current market penetration is low. I refer to  $\alpha_n$  and  $\beta_n$  as the macroeconomic and microeconomic returns to market size parameters, respectively.  $\gamma_n \in \mathbb{R}_{++}$  represents the degree of diminishing returns to advertising to new customers. The higher  $\gamma_n$ , the fewer additional new customers are reached by each additional unit of search advertising. Finally,  $\psi_n \in \mathbb{R}_{++}$  is the efficiency of advertising to new customers

Similarly, the marginal effect of retention effort is increasing in the total number of old customers,  $mL_j$ , and decreasing in the fraction of old customers the firm successfully retains,  $o / m$ :

$$o'_j(r) = \psi_o L_j^{-\alpha_o} m^{-\beta_o} \left( \frac{m - o_j(r)}{m} \right)^{\gamma_o}. \quad (15)$$

The parameters  $\alpha_o$ ,  $\beta_o$ ,  $\gamma_o$ , and  $\psi_o$  have similar interpretations to their counterparts above. Note though, that microeconomic returns to market size wax as a firm's market penetration, and thus the pool of old customers who can be retained, grows. Differences between parameters of (14) and (15) allow for the possibility that advertising to old customers works differently than advertising to new customers. For example, it may be that the macroeconomic market size effect is less pronounced ( $\alpha_o > \alpha_n$ ) because advertising to current customers is more analogous to contacting them individually one after another than to mass advertising on the radio or television. It might also be the case that returns to advertising to current customers diminish less rapidly ( $\gamma_o < \gamma_n$ ). Indeed, when I calibrate the model's parameters so that it matches the facts described in section 2, I find precisely these differences.

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<sup>6</sup>I assume that firms can direct their advertisements to these two segments of the market's population separately; firms know the identities of their current customers and can advertise directly to them. Alternatively, one could assume that firms can target advertising to their current customers but must advertise across the entire population to reach new customers. What is key is that a firm with more current customers must advertise more to keep them.

### 3.4 Market penetration cost dynamics

Solving the differential equations (14) and (15) yields the costs of attracting  $n$  new customers and retaining a  $o$  old customers, respectively:

$$s_j(m, n) = \frac{L_j^{\alpha_n} (1-m)^{\beta_n}}{\psi_n (1-\gamma_n)} \left[ 1 - \left( \frac{1-m-n}{1-m} \right)^{1-\gamma_n} \right], \quad (16)$$

$$r_j(m, n) = \frac{L_j^{\alpha_o} m^{\beta_o}}{\psi_o (1-\gamma_o)} \left[ 1 - \left( \frac{m-o}{m} \right)^{1-\gamma_o} \right]. \quad (17)$$

Although these expressions bear more than a passing resemblance to the market penetration costs in Arkolakis (2010), they depend not only on the number of customers a firm attracts or retains, but also on the firm's current market penetration. Thus, as a firm builds its customer base over time, its search and retention costs change.

The advertising cost functions (16) and (17) have several key properties that allow the model to account for both the cross-sectional and life-cycle facts about exporters documented above in section 2. First, the marginal costs of attracting the first new customer and retaining the first old customer are strictly positive regardless of a firm's current market penetration:

$$s_{j,n}(m, 0) > 0 \forall m, \quad r_{j,o}(m, 0) > 0 \forall m, \quad (18)$$

where  $s_{j,n}$  and  $r_{j,o}$  denote partial derivatives of the advertising cost functions with respect to their first arguments. This implies that sufficiently unproductive firms may find the cost of attracting the first new customer or retaining the first old one prohibitive. As in Arkolakis (2010), this property delivers an endogenous extensive margin of trade; potential entrants with sufficiently low productivities will choose not to enter. In this dynamic context, this property also delivers endogenous exit: incumbent firms with low enough productivities will also opt not to retain any of their customers or attract any new ones. Moreover, if the marginal cost of attracting the first new customer for a potential entrant,  $s_n(0, 0)$ , exceeds the marginal cost of retaining the first old customer for a typical incumbent, the model will generate exporter hysteresis: the average entrant will have a higher productivity than the average incumbent.

Second, the marginal costs of attracting new customers and retaining old ones are increasing, and it is impossible to attract all potential new customers and retail all old customers:

$$s_{j,nn}(m, n) > 0 \forall n, m, \quad r_{j,oo}(m, o) > 0 \forall o, m, \quad (19)$$

$$\lim_{n \rightarrow 1} s_{j,n}(m, n) = \lim_{o \rightarrow 1} r_{j,o}(m, o) = \infty \forall m. \quad (20)$$

Here,  $s_{j,nn}$  and  $r_{j,oo}$  denote second partial derivatives. These properties, which are also similar to properties

from Arkolakis (2010), imply that more productive firms will attract more new customers and retain more of their old customers, but even the most productive firms will not fully penetrate the market or retain all of their customers. Note that this implies that all firms experience customer turnover, even firms with growing customer bases.

Third, the marginal cost per customer of attracting new customers is increasing in current market penetration:

$$s_{j,nm}(m, n) > 0 \forall n, m, \quad (21)$$

where  $s_{j,nm}$  denotes the cross partial derivative of the per-customer search advertising cost function. This implies that new exporters with low market penetration rates will attract more new customers than older exporters with higher market penetration rates. This property captures the idea of “low-hanging fruit:” firms first attract the customers who are easiest to reach, and then slow down as attracting new customers gets harder. As a result, new exporters are smaller than incumbents but experience catch-up growth.

Fourth, the marginal cost of retaining old customers is decreasing in current market penetration:

$$r_{j,om}(m, o) < 0 \forall o, m. \quad (22)$$

This implies that advertising to retain old customers is more effective for firms with larger customer bases. Together with the first property, this implies the cost of retaining the first customer is decreasing in market penetration, which implies that smaller (i.e. younger) firms are more likely to exit.

### 3.5 Firm’s problem

The problem of a firm is to choose advertising efforts in each foreign market to maximize the present value of the profits from exporting, taking its productivity, demand shock, and current market penetration in each of these markets as given. Because firms’ production technologies have constant returns to scale and advertising is independent across destinations, the firm’s problem can be solved destination by destination. Breaking the firm’s problem into two stages facilitates the characterization of its solution. In the first stage, the firm takes as chooses search and retention efforts to minimize its total cost of exporting taking as given how much it will increase or decrease its market penetration. In the second stage, the firm chooses how much to increase its market penetration taking its first-stage choice as given.

### 3.5.1 Export cost minimization

For a firm with current market penetration  $m$  that wishes to expand (or perhaps shrink) its market penetration to  $m'$  next period, the total cost of exporting is given by

$$f_j(m, m') = \min_{n \in [0, 1-m], o \in [0, m]} \{s_j(m, n) + r_j(m, o)\} \text{ subject to } n + m = m'. \quad (23)$$

I use  $n_j(m, m')$  and  $o_j(m, m')$  to denote the optimal policy functions for customer attraction and retention, respectively. Using properties (18)–(20), the solution to this problem can be characterized as follows:

- For entrants, who have no old customers to retain, the advertising cost is equal to the attraction cost:  $a_j(0, m') = s_j(0, m')$ ,  $n_j(0, m') = m'$ , and  $r_j(0, m') = 0$ .
- If the firm's market penetration increases and the marginal cost of attracting the last new customer is lower than the marginal cost of retaining the first old customer, then no old customers should be retained: If  $m' > m$  and  $s_{j,n}(m, m') < r_{j,o}(m, 0)$ , then  $n_j(m, m') = m'$  and  $o_j(m, m') = 0$ .
- If the firm's market penetration shrinks and the marginal cost of retaining the last old customer is lower than the marginal cost of attracting the first new customer, then no new customers should be attracted: If  $m' < m$  and  $r_{j,o}(m, m') < s_{j,n}(m, 0)$ , then  $n_j(m, m') = 0$  and  $o_j(m, m') = m'$ .
- Otherwise, the marginal attraction and retention costs are equal at the optimum:  $s_{j,n}(m, n) = r_{j,o}(m, o)$ .

Figure 6 provides a graphical depiction of the solution to this problem in the calibrated model for entrants (blue lines), firms with low market penetration rates (red lines), and firms with high market penetration rates (green lines).

Panels (a) and (b) show that firms with higher market penetration rates attract fewer new customers and retain more old customers than firms with lower market penetration rates. This is true in both absolute and relative terms; retained old customers make up a larger share of next period's customer base for firms with higher current market penetration rates. In fact, firms with high current market penetration rates may not attract any new customers at all if their choose sufficient low market penetration rates in the next period.

Panel (c) shows that the overall exporting cost is highest for entrants and lowest for firms with high market penetration rates. Formally, the exporting cost is decreasing in a firm's current market penetration:

$$f_{j,m}(m, m') < 0, \forall m, m'. \quad (24)$$

This finding mirrors a common result in the literature on sunk-cost models, in which the cost of entering a foreign market is higher than the cost of continuing to serve it. Here, it implies that a firms derive two benefits from expanding their customer bases: increased sales in the present and reduced exporting costs in the future.

Panel (d) shows that the marginal advertising cost is also highest for entrants and lowest for firms with high market penetration rate. In other words, the marginal advertising cost is also decreasing in a firm's current market penetration:

$$f_{j,m'm}(m, m') < 0, \forall m, m'. \quad (25)$$

This property of the exporting cost function is inherited from property (22) of the retention cost function and the fact that firms with higher market penetration rates do more customer retention. This property is what generates “new exporter dynamics” in which entrants export less than incumbents but catch up over time. Panel (d) also shows that the marginal cost of attracting or retaining the very first customer is always positive:

$$f_{j,m'}(m, 0) > 0, \forall m. \quad (26)$$

This property, which is inherited from property (18) of the attraction and retention cost functions, implies that firms with sufficiently low productivities and/or demand shocks will exit endogenously (if  $m > 0$ ) or not enter (if  $m = 0$ ).

Figure 7 shows how the costs of exporting vary across destinations. The blue (entrant) and green (incumbent) solid lines show the total and marginal exporting costs for an “easy” destination with a large population and high income per capita. The red (entrant) and purple (incumbent) dashes lines show the total and marginal costs for a “hard” destination with a small population and low income per capita. I have normalized the exporting cost for each destination by its purchasing power  $L_j Y_j$  to illustrate how the payoff from exporting compares to the cost. Measured relative to purchasing power in this way, exporting is more expensive on the whole and at the margin in the harder destination. This implies that in equilibrium, firms will accumulate fewer customers in harder markets as in Arkolakis (2010). And because the marginal cost of the first customer,  $f_{j,m'}(m, 0)$ , is higher in the harder destination, fewer firms will enter that market to begin with.

### 3.5.2 Optimal market penetration

Once the firm has determined the most cost-effective way to increase or decrease its market penetration, it chooses how much it should do so in order to maximize the present discounted value of the profits from exporting:

$$V_j(x, z, m) = \max_{m' \in [0,1]} \left\{ \pi_j(z, m') - f_j(m, m') + \frac{\delta(x)}{1+r} \mathbb{E} [V_j(x, z', m') | x, z] \right\} \quad (27)$$

The parameter  $r$  governs the rate at which the firm discounts future profits. This formulation of the problem is virtually identical to the Bellman equations in sunk-cost models of exporting like Das et al. (2007), Alessandria and Choi (2007), and Alessandria et al. (2014) in which the cost of exporting depends on a firm's current status as an exporter. The only difference is that export status is continuous, rather than binary, capturing both extensive and intensive margins. I use  $m'_j(x, z, m)$  to denote the optimal policy function at this stage.



Using the envelope theorem, the solution to this problem is characterized by the following inequality:

$$f_{j,m'}(m, m') \geq \tilde{\theta} L_j Y_j \tau_j^{1-\theta} (xz)^{\theta-1} - \frac{\delta(x)}{1+r} \mathbb{E} [f_{j,m}(m', m'') | x, z], \quad (28)$$

where  $m'$  and  $m''$  are shorthand for  $m'_j(x, z, m)$  and  $m''_j(x, z', m'_j(x, z, m))$ , respectively. The left-hand side of this expression is the marginal cost of exporting. The first term on the right-hand side is the marginal increase in flow profits the firm gains from increasing its market penetration. The second term on the right hand side is the expected change in the cost of exporting next period. Note that property (24) implies that this term is positive: increasing market penetration today reduces the cost of exporting tomorrow. If this condition holds with equality, the firm chooses  $m'$  to equate the marginal cost of exporting with the marginal benefit. Property (25) implies that firms with higher market penetration rates at the beginning of the period will choose higher market penetration rates at the end of the period, i.e., the policy function is upward-sloping:  $m'_{j,m}(x, z, m) > 0$ . This implies that firms gradually build up their customer bases over time.

If, on the other hand, the marginal cost of attracting or retaining the very first customer,  $f_{j,m'}(m, 0)$ , exceeds the marginal benefit, the firm will exit (if  $m > 0$ ) or not enter (if  $m = 0$ ). Entry is characterized by threshold,  $\underline{z}(x)$  such that firms with demand shocks below this threshold will choose not to enter:

$$f_{j,m'}(0, 0) \geq \tilde{\theta} L_j Y_j \tau_j^{1-\theta} (x\underline{z}(x))^{\theta-1} - \frac{\delta(x)}{1+r} \mathbb{E} [f_{j,m}(0, m'') | x, z]. \quad (29)$$

The entry threshold is decreasing in a firm's productivity:  $\underline{z}'(x) < 0$ . Thus, high-productivity firms are more likely to exit than low-productivity firms. Exit is characterized by a threshold  $\underline{m}(x, z)$  such that firms with market penetration below this threshold will choose to exit:

$$f_{j,m'}(\underline{m}(x, z), 0) \geq \tilde{\theta} L_j Y_j \tau_j^{1-\theta} (xz)^{\theta-1} - \frac{\delta(x)}{1+r} \mathbb{E} [f_{j,m}(0, m'') | x, z]. \quad (30)$$

Property (25) implies that the exit threshold is decreasing in productivity and demand:  $\underline{m}_x(x, z) < 0$ ,  $\underline{m}_z(x, z) < 0$ . This means that firms with lower market penetration rates are more likely to exit than firms with higher market penetration rates. Because the policy function is increasing, this means that firms with shorter tenures in a market are more likely to exit as documented by Ruhl and Willis (2017), Alessandria et al. (2014), and others. Conversely, it also means that a firm with a demand shock below the entry threshold may choose not to exit if it begins the period with a high enough market penetration rate. Thus, a firm that enters when its demand is high may accumulate a large enough customer base that it is profitable to remain in the market after receiving a poor demand shock that would have precluded it from entering initially. Also, note that although multilateral exit can also occur exogenously through death, bilateral exit only occurs endogenously because death shocks are independent of firms' bilateral demand shocks.

Figure 8 illustrates how the features of the model work together to generate realistic exporter dynamics.

Consider a firm born in period 0 with zero customers in market  $j$  and a high enough demand shock,  $z_{hi}$ , to warrant entering that market. Panel (a) shows how its optimal market penetration choice as a new entrant,  $m_1$ , is determined. It is shown in the figure as the intersection of the firm's marginal benefit, the upper horizontal dotted line labeled  $\tilde{\pi}_j(xz_{hi})^{\theta-1} + \beta\mathbb{E}[f_{j,m}|z_{hi}]$ , and the entrant's marginal cost curve, the solid blue line labeled  $f_{j,m'}(0, \cdot)$ . Panel (b) shows the firm's policy function as the solid blue line labeled  $m'_j(x, z_{hi}, \cdot)$ ; the firm's choice in this period is the point  $(0, m_1)$  located on this line.

In period 1, the firm's marginal cost curve shifts outward to the dashed red line in panel (a) labeled  $f_{j,m'}(m_1, \cdot)$  due to property (25). The firm's optimal market penetration choice in this period,  $m_2$ , is given by the intersection of this new marginal cost curve and the firm's marginal benefit.<sup>7</sup> The firm's choice in period 1 is shown in panel (b) as the point  $(m_1, m_2)$  on the firm's policy function. In period 2, the firm's marginal cost curve shifts outward again, to the green dash-dotted line labeled  $f_{j,m'}(m_2, \cdot)$  in panel (a). Suppose, however, that the firm receives a bad demand shock,  $z_{lo}$ , such that the marginal benefit of exporting is now lower than the marginal cost (shown by the lower horizontal dotted line in panel (a) labeled  $\tilde{\pi}_j(xz_{lo})^{\theta-1} + \beta\mathbb{E}[f_{j,m}|z_{lo}]$ ). Instead of continuing to expand, the firm decides to exit. The red dashed line labeled  $m'_j(z, z_{lo}, \cdot)$  in panel (b) shows the policy function associated with this lower level of demand; the firm's decision to exit is shown as the point  $(m_2, 0)$  on this curve.

Now suppose instead that the firm keeps its higher demand shock instead of receiving the bad one. In this case, its optimal market penetration choice,  $m_3$ , is shown in panel (a) as the intersection of its current marginal cost curve,  $f_{j,m'}(m_2, \cdot)$ , and its original marginal benefit. This choice is shown in panel (b) as the point  $(m_2, m_3)$  on the original policy function (the solid blue line). Note that the firm's policy function at  $m_3$  is positive: this level of market penetration is high enough that the firm will not longer choose to exit if it receives the bad demand shock. This illustrates how the model generates higher exit rates among smaller, younger exporters.

### 3.6 Aggregation and equilibrium

The final piece of the model is a law of motion that describes how the distribution of exporters evolves over time. Let  $\Psi_j(x, z, m)$  denote the joint distribution of productivities, demand shocks, and market penetration rates in market  $j$ . This distribution evolves according to the law of motion

$$\Psi'_j(\mathcal{X} \times \mathcal{Z} \times \mathcal{M}) = \int_{\mathbb{R}^2_{++} \times [0,1]} Q_j(x, z, m, \mathcal{X} \times \mathcal{Z} \times \mathcal{M}) d\Psi_j(x, z, m), \quad (31)$$

where  $\mathcal{X}$ ,  $\mathcal{Z}$ , and  $\mathcal{M}$  denote typical subsets of  $\mathcal{X}$ ,  $\mathcal{Z}$ , and  $[0, 1]$ , respectively, and  $Q_j(x, z, m, \mathcal{X} \times \mathcal{Z} \times \mathcal{M})$  is the probability that a firm with productivity  $x$ , demand shock  $z$ , and customer base  $m$  transits to a state in

<sup>7</sup>The graph shows a single line for the firm's marginal benefit, when in fact it shifts upwards over time as the firm's rising market penetration lowers the marginal exporting cost due to property (24). However, in the calibrated model these shifts are small relative to the scale of the marginal cost curve; the figure shows the actual dynamics of a firm in the calibrated model

the set  $\mathcal{X} \times \mathcal{Z} \times \mathcal{M}$ . This transition function is given by

$$Q_j(x, z, m, \mathcal{X} \times \mathcal{Z} \times \mathcal{M}) = \delta(x) \int_{\mathcal{Z} \times \mathcal{X}} \mathbb{1}_{\{m'_j(x, z, m) \in \mathcal{M}\}} dG(x', x) dH(z', z) \quad (32)$$

$$+ (1 - \delta(x)) \int_{\mathcal{X} \times \mathcal{Z}} \mathbb{1}_{\{0 \in \mathcal{M}\}} d\bar{G}(x') d\bar{H}(z').$$

The first term on the right-hand side is probability that a firm survives, chooses a new customer base in the set  $\mathcal{M}$ , draws a productivity shock in the set  $\mathcal{X}$ , and draws a demand shock in the set  $\mathcal{Z}$ . The second term is the probability that a firm dies and is replaced by a new firm with productivity in the set  $\mathcal{X}$  and demand shock in the set  $\mathcal{Z}$ .

A stationary equilibrium is: (i) a collection of export cost policy functions,  $(n_j(m, m'), o_j(m, m'), f_j(m, m'))_{j=1}^J$ , that solve the cost minimization problem (23); a collection of value functions and market penetration policy functions,  $(V_j(x, z, m), m'_j(x, z, m))_{j=1}^J$ , that solve the firm's dynamic problem (27); and (iii) a collection of distributions,  $(\Psi_{j,t})_{j=1}^J$ , that satisfy the law of motion (31). In my quantitative analysis, I solve for transition dynamics following permanent and temporary changes in trade costs as well as stationary equilibria; a transition equilibrium is a sequence of sets described above that satisfy the relevant conditions at each point in time.

## 4 Calibration

I calibrate the model's parameters so that it matches the variation in exporter performance across destinations documented in section 2.1 using an indirect inference approach. The calibrated model reproduces the targeted moments as well as the non-targeted facts about how exporters' performance varies with time in a market and across destinations documented in sections 2.2–2.3. After calibrating and validating the model, I explore how the costs of exporting that firms choose in equilibrium vary with time in a market and across destinations.

### 4.1 Procedure

The first step in my calibration procedure is to choose a set of destinations and assign values to their characteristics,  $L_j$ ,  $Y_j$ , and  $\tau_j$ . I use the same 63 destinations in the Brazilian customs data that analyze in section 2. I draw values for their characteristics from the CEPII Gravity database. Population,  $L_j$ , and GDP per capita,  $Y_j$ , are used without modification. My measure of trade costs,  $\tau_j$ , are the residuals from a simple gravity regression of exports on source and destination populations and incomes per capita; they capture everything from tariffs to distance to language barriers.

The second step is to calibrate the parameters that govern the distribution of firms' exogenous types and the cost of exporting. I assume that demand follows a standard AR(1) process in logs with persistence

$\rho_z$  and innovation dispersion  $\sigma_z^2$ . I assume that productivity is unconditionally distributed log-normally with variance  $\sigma_x^2$ , and each period firms retain their productivities with probability  $\rho_x$  and draw new ones with probability  $1 - \rho_x$ .<sup>8</sup> Following Alessandria et al. (2014), I parameterize the death rate as  $1 - \delta(x) = \max(0, \min(e^{-\delta_0 x} + \delta_1, 1))$ . With these parameterizations, there are 16 parameters that must be calibrated:  $\rho_x, \sigma_x, \rho_z,$  and  $\sigma_z$  govern the distributions of the exogenous state variables;  $\delta_0$  and  $\delta_1$  govern survival;  $\alpha_n, \beta_n, \gamma_n,$  and  $\psi_n$  govern the cost of attracting new customers;  $\alpha_o, \beta_o, \gamma_o,$  and  $\psi_o$  govern the cost of retaining old customers;  $\theta$  governs the elasticity of substitution between varieties; and  $r$  governs the rate at which firms discount future profits. Following Ruhl and Willis (2017) and Alessandria et al. (2014). I set  $\theta$  externally to 5, a common value in the literature that implies a trade elasticity of 4 in the absence of firm-level responses. I set the discount rate  $r$  externally to match the average Brazilian real interest rate during 2000-2005 of 10%.<sup>9</sup> This leaves me with 14 parameters whose values must be jointly determined.

I use an indirect inference strategy to find values of these parameters that minimize the distance between moments in the Brazilian customs data and moments in simulated data generated by the model. Specifically, for each of the six measures of exporter performance discussed in section 2.1, I target the cross-destination average shown in table 1 and the coefficients on population, income per capita, and trade barriers shown in table 2. Additionally, I target a multilateral export participation rate of 26%.<sup>10</sup> For each candidate parameter vector, I use the model to simulate a panel of firms and calculate these moments by applying the same processing and analysis that I apply to the Brazilian customs data. I then search over the parameter space to find the vector of parameters that minimizes the mean squared error between the simulated moments and actual moments, where each moment is weighted by the inverse of its standard error. To ensure that the estimated parameter vector is a global minimum, I break the parameter space into increasingly small subspaces, use a stochastic population-based global optimization method in each subspace, and “polish off” each subspace’s best candidate parameter vector using a simplex-based method. Essentially, my approach follows the Subplex method (Rowan, 1990) but adds a stochastic search in each subspace.

There are a total of 25 target moments (the average and three slope coefficients for each of the six statistics, plus the overall export participation rate). The 14 estimated parameters are therefore technically overidentified, but the several of the target moments are correlated. Looking at the average number of other destinations served, for example, the cross-destination mean is negatively correlated with the three slope coefficients because exporters in the most popular markets serve only a few other destinations; increasing the magnitude of the slope coefficients for this statistic also raises the overall mean. Consequently, an exactly-identified estimation strategy would be problematic, whereas my strategy ensures there is sufficient independent variation in the target moments to pin down all of the parameters. I have experimented with

<sup>8</sup>This approach is similar to a standard log-normal AR(1) process, but is more computationally tractable because an exporter’s continuation value conditional on drawing a new productivity is independent of its current productivity. It is commonly used in models with Pareto-distributed productivities (see, e.g., Buera et al., 2011)

<sup>9</sup>Brazilian real interest rates were high and volatile during the 1980s and 1990s and then declined after the Brazilian currency was allowed to float in 1999. The 10% figure I use is almost identical to the value used by Ruhl and Willis (2017) for Colombia.

<sup>10</sup>I do not have data on non-exporting Brazilian firms, so I rely on the estimate of Ruhl and Willis (2017) for Colombia.

different sets of moments, e.g. including all the facts about how export performance varies with time in a market from section 2.2,

Each of the target moments affects some parameters more than others. The cross-destination averages of the top-5 share and the number of other destinations served pin down the variances of the productivity distribution and demand shock,  $\sigma_x$  and  $\sigma_z$ . The overall export participation rate determines the level of the new-customer attraction cost,  $\psi_n$ , while the average exit rate influences the level of the old-customer retention cost,  $\psi_o$ , and the minimum death probability,  $\delta_1$ . The average exit rate of entrants relative to that of incumbents affects the persistence of the demand shock,  $\rho_z$ , and the sensitivity of the exit rate to productivity,  $\delta_0$ . The slope coefficients of the exit rate, number of other destinations served, and relative exit rate of entrants play dominant roles in identifying the returns to market size in attracting new customers and retaining old ones,  $\alpha_n$  and  $\alpha_o$ . Finally, the average and slope coefficient of the relative entrant size and the slope coefficient of the top-5 share jointly pin down the convexity parameters of the attraction and retention costs,  $\gamma_n$  and  $\gamma_o$ .

## 4.2 Parameter values

Table 5 lists the parameter estimates resulting from the procedure described above.<sup>11</sup> Panel (a) shows the parameters that govern the distribution and evolution of firms' exogenous characteristics. The variance of multilateral productivity shocks is somewhat lower than that estimated in models of multilateral exporter dynamics (see, e.g., Ruhl and Willis, 2017; Alessandria and Choi, 2014; Alessandria et al., 2014), but the persistence of these shocks is higher. However, demand shocks are substantially less persistent than productivity shocks, and the product of productivity and demand,  $x \times z_j$ , exhibits similar dispersion and persistence to other studies' productivity processes. The variance of the productivity distribution on top of the variance in the demand shock process is needed to capture the high concentration of exports among the largest exporters in the average destination and the fact that most firms have relatively small portfolios of export destinations, while the lower persistence of demand helps account for variation in exit rates across destinations. The survival function parameters, shown in panel (b), match almost exactly the estimates of Alessandria et al. (2014), who use business dynamics data in which firm creation and death can be directly observed.

The parameters of the advertising cost functions are shown in panels (c) and (d). The macroeconomic return to market size is significantly larger in attracting new customers than in retaining old ones:  $\alpha_n < \alpha_o$ . This captures the idea that advertising to current customers is more akin to contacting them individually, one by one, whereas advertising to new customers is more like advertising on the radio or television en masse. Conversely, the microeconomic return to market size is larger in advertising to old customers:  $\beta_o < \beta_n$ . The new-customer attraction cost function is substantially more convex than the old-customer retention cost

<sup>11</sup>Reporting standard errors is not feasible due to the computational burden of solving and simulating the model, but also due to the fact that the objective function exhibits discrete jumps and other non-differentiabilities.

function:  $\gamma_n > \gamma_o$ . This indicates that it is harder to attract large blocks of new customers than to retain large blocks of old ones. Finally, the level parameter is higher for customer attraction than for retention:  $\psi_n > \psi_o$ . Taken at face value, this would seem to suggest that acquiring new customers is cheaper than retaining old ones, which would contrast sharply with the large startup costs in standard sunk-cost models like Das et al. (2007) and Alessandria and Choi (2007) required to generate realistic export participation and turnover. However, as I show in section 4.4, when we examine more closely the exporting costs that firms choose to incur in equilibrium, we see that this is not exactly the case.

### 4.3 Performance on targeted and non-targeted moments

The calibrated model closely replicates the targeted moments. The first row in panel (b) of table 1 shows the means of the 6 cross-sectional and dynamic measures of exporter performance discussed in section 2.1 in the simulated data. The mean top-5 share, average number of destinations served, exit rate, relative entrant size, and relative entrant exit rate are all very close to the means observed in the Brazilian data. The mean number of exporters is two-thirds higher in the simulated data, but this moment is weighted substantially less than the other measures' means due to its relatively high standard error. Panel (b) of table 2 shows the associations between destination characteristics and these measures of exporter performance in the simulated data. All coefficients but one (the effect of population size on the exit rate) have the correct sign, and of the former all but one (the effect of population on the relative exit rate of entrants) have the correct magnitude. Figure 5, which provides a visualization of how the model fares in capturing the stylized facts described in section 2.1, shows that the model reproduces all of the expected relationships between export participation and the 6 cross-sectional and dynamic measures, both qualitatively and quantitatively.

The model also reproduces virtually all of the other facts documented in 2 that were not targeted in the calibration. The last four rows in panel (b) of table 1 show how the model performs in reproducing other summary statistics for the measures discussed in section 2.1. The model generates a similar amount of variation across destinations as in the data for the number of exporters, the top-5 share and the relative size of entrants, and about twice as much variation for the overall exit rate and the relative exit rate of entrants.

Figure 2 shows that the model closely replicates the facts about how export performance varies with time in a market described in section 2.2. In both model and data, the most successful exporters exhibit the strongest growth in sales over the duration of their export spells and sell more upon entry than less successful exporters; sales drop in the period immediately preceding exit; and the differences in sales dynamics between more- and less-successful exporters are more pronounced in easy destinations than in hard ones. Figure 3 shows that the model qualitatively captures the fact that the likelihood of exit falls with time in a market, but does not generate as much of a decline as in the data and exhibits a larger difference between easy and hard destinations.

Figure 4 shows that the distribution of exporters by destinations served in the calibrated model is close to the empirical distribution, as is the distribution of exports. This confirms that the model captures how the cross sections of exporters varies across destinations. Table 4 shows that the calibrated model also matches the propensity of multi-destination firms to exit more frequently from their least-important destinations. Finally, 5 shows that the model predicts similar patterns of sales and exit rates across destinations within individual exporters' portfolios as observed in the data. In the model, as in the data, exporters with larger portfolios sell more in their top-ranked destinations relative to single-destination exporters, but sell less in their least important destinations. Similarly, exporters with larger portfolios are less likely to exit from their most important destinations as compared to single-destination exporters, and exporters with 4 or fewer destinations have about the same exit rate in their lowest-ranked destinations as single-destination exporters. Together, these tables and figures show that the model accurately captures the facts about how export performance varies within exporters' destination portfolios discussed in section 2.3.

#### 4.4 Equilibrium exporting costs

To illustrate how exporting costs vary in equilibrium over firms' tenures in different destinations, I estimate how time in a market affects exporting costs, both in levels and relative to profits, following the Fitzgerald et al. (2016) approach described in section 2.2:

$$\log f(m_{i,j,t}, m_{i,j,t+1}) = \alpha + \sum_{m=1}^6 \sum_{n=1}^{m-1} \beta_{m,n} \mathbb{1}_{\{\text{duration}_{i,j}=m\}} \mathbb{1}_{\{\text{years in market}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t}, \quad (33)$$

$$\frac{f(m_{i,j,t}, m_{i,j,t+1})}{\pi_j(x_{i,j,t}, x_{i,j,t}, m_{i,j,t})} = \alpha + \sum_{m=1}^6 \sum_{n=1}^{m-1} \beta_{m,n} \mathbb{1}_{\{\text{duration}_{i,j}=m\}} \mathbb{1}_{\{\text{years in market}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t}. \quad (34)$$

As before, I split destinations into hard and easy groups based on export participation, and the reference group is firms that export for only one year before exiting. Note though, that I exclude the last year of a firm's export spell (i.e. when  $m = n$ ) because firms that choose to exit endogenously pay zero export costs. Panels (a) and (b) of figure 10 report the estimated effects of time in a market on the level of exporting costs from specification (33). Exporting costs are approximately constant over the duration of an export spell in hard destinations, but do in fact rise with time in a market in easy destinations. Panels (c) and (d), however, which report the estimated effects of time in a market on the ratio of exporting costs to profits from specification (34), tell a different story. Measured relative to profits, exporting costs are highest at the beginning of an export spell, and decline more sharply in easy destinations than in hard ones. Together, these results line up nicely with calibrated models of exogenous new exporter dynamics like Ruhl and Willis (2017) and Alessandria et al. (2014), which typically feature startup costs that are similar to continuation costs when measured in levels, but substantially higher when measured relative to profits.

What's new here is that exporting costs also vary endogenously across firms within each destination.

Figure 10 shows that in both groups of destinations, more successful exporters (those with longer export spells in a given destination) pay higher exporting costs than less successful exporters after controlling for spell duration and tenure, although the differences are larger in easy destinations, mirroring the results for sales shown in figure 2. To dig more deeply into cross-firm variation in exporting costs and to analyze how individual firms' exporting costs vary across destinations, I follow the approach from section 2.3 and estimate the effect of a destination's rank within an exporter's portfolio on the cost that exporter pays to access that destination, both in levels and relative to profits:

$$\log f(m_{i,j,t}, m_{i,j,t+1}) = \alpha + \sum_{m=1}^{10} \sum_{n=1}^m \beta_{m,n} \mathbb{1}_{\{\text{num. dests.}_{i,j}=m\}} \mathbb{1}_{\{\text{dest. rank}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t}, \quad (35)$$

$$\frac{f(m_{i,j,t}, m_{i,j,t+1})}{\pi_j(x_{i,j,t}, x_{i,j,t}, m_{i,j,t})} = \alpha + \sum_{m=1}^{10} \sum_{n=1}^m \beta_{m,n} \mathbb{1}_{\{\text{num. dests.}_{i,j}=m\}} \mathbb{1}_{\{\text{dest. rank}_{i,j,t}=n\}} + f_j + f_t + \epsilon_{i,j,t}. \quad (36)$$

Again, the reference group is the set of firms that serve only destination  $j$ . Panel (a) of figure ?? reports the estimated effects of destination rank on the level of exporting costs from specification (35). Exporters with the largest destination portfolios pay the highest exporting costs, especially in higher-ranked destinations. Firms that serve 10 or more destinations, pay about 4 times more to export to their top destinations than firms that serve those destinations only, and even firms that sell to only 2 destinations pay twice as much as single-destination exporters. Export costs fall with destination rank, however, and firms with 9 or fewer destinations in their portfolios (the vast majority) actually pay less to export to their least important destinations than firms that serve those destinations alone. Panel (b) of figure ??, which reports the results from specification (36), shows that these results reverse when export costs are measured relative to profits. Firms with larger destination portfolios have lower export cost/profit ratios, especially in their highest-ranked destinations, and these ratios rise as destination rank falls. In brief, these results show that high-productivity and/or high-demand firms pay higher costs to export, but these costs are low relative to the large profits these firms earn from exporting.

## 5 Aggregate implications

A common theme in the trade dynamics literature is that micro matters for macro: firm-level responses drive the dynamics of aggregate trade flows in response to permanent trade policy reforms and temporary shocks. Here, I conduct three experiments using the calibrated model to study how differences in exporter dynamics across destinations generate differences in bilateral trade dynamics. First, I conduct a standard exercise in the trade literature: tracing out transition dynamics in response to a permanent reduction in trade costs. Next, I analyze the transition dynamics that follow a temporary real exchange rate shock. Third, I analyze the implications of trade policy uncertainty. In addition, I corroborate the model's aggregate implications by using the Brazilian customs data to analyze the bilateral trade dynamics that followed the depreciation



of Brazil's real exchange rate in 1999.

## 5.1 Permanent trade policy reforms

In my first experiment, I use the calibrated model to analyze how bilateral trade flows respond in the short and long run to a permanent, unanticipated 10% reduction in iceberg trade costs. I solve for the transition dynamics that follow this change in each of the destinations in the Brazilian customs data, and then break the resulting time series into two groups: destinations in the top 10% of export participation (easy destinations), and destinations on the bottom 50% (hard destinations). I adopt the following timing to make the different forces at work as transparent as possible. In period 0, the model is in its initial steady state. In period 1, trade costs fall after firms have made their market penetration decisions, so trade rises only because of the price elasticity of demand. Thus, the period-1 trade elasticity in all destinations is  $\theta - 1 = 4$ , the elasticity that would obtain in a model without any firm-level adjustments at all. In period 2, firms begin to adjust their market penetration rates, entering and expanding due to the increase in demand, and the trade elasticity begins to rise.

Figure 11 shows how the number of exporters, the average market penetration rate, and total bilateral exports evolve over time in hard versus easy destinations in response to this trade reform. In the long run, export participation and market penetration both respond substantially more in hard destinations than in easy destinations. This is closely related to observations by Arkolakis (2010), Eaton et al. (2011), Kehoe and Ruhl (2013), and others that the least-traded products respond the most to trade reforms. The result of this “new-goods margin” or “least-traded-products” effect is that harder destinations have long-run trade elasticities more than 20 percent greater than easy destinations. In fact, the long-run trade elasticity in easy destinations is only slightly above  $\theta - 1$ . This is because exports are highly concentrated among the largest firms in these destinations, and “the largest firms in a market grow at a positive rate that (asymptotically) depends only on the price elasticity of demand.” (Arkolakis, 2010).

In the short run, trade takes several years to converge to its new higher level as new firms enter and others build up their customer bases. The export participation rate converges in about 6 years, while the average market penetration rate takes about 10 years. In hard destinations, where exports are more evenly distributed across firms and these firm-level adjustments are more pronounced, the trade elasticity also takes about 10 years to converge. In easy destinations, by contrast, trade flows converge almost immediately because firm-level adjustments are quantitatively unimportant.

## 5.2 Temporary real exchange rate depreciation

In my second experiment, I use the calibrated model to analyze how trade responds to a temporary 10% increase in the real exchange rate. Here, demand in each destination rises unexpectedly by 10%, and then

this boost decays geometrically over time by 10% each period. Formally, the demand curve for a firm's product in each period  $t$  shifts outward to  $c_{j,t}(z, p) = L_j Y_j P_t^\theta (p/z)^{-\theta}$  where  $P_t = e^{0.1 \times 0.9^t}$ . The timing is the same as in the previous experiment: the shock hits in period 1, and then firms start adjusting in period 2.

Figure 12 shows how export participation, market penetration, and exports evolve over time in response to this shock in hard versus easy destinations. As in the previous exercise, trade rises by the same amount in all destinations on impact—the short-run trade elasticity of  $\theta - 1$ —because firms have not yet changed their market penetration decisions. In the following periods, export participation and market penetration respond more strongly in hard destinations than in easy destinations—the same “least-traded-products effect” that makes hard destinations to respond more to permanent trade reforms. In turn, this leads to more hysteresis in hard destinations: export participation, market penetration, and overall exports are more persistent than the real exchange rate shock itself. In easy destinations, by contrast, there is little hysteresis: trade follows the path of the real exchange rate almost exactly.

### 5.3 Trade policy uncertainty

In my last experiment, I analyze how trade policy uncertainty affects export participation in the model. Here, I repeat the first exercise with a twist: after trade costs fall, there is a 50% chance each period that they will rise again. This captures in a stylized way the kind of uncertainty that surrounded U.S. trade policy towards China during 1980-2000 as studied by Handley and Limão (2017), Pierce and Schott (2016), and Alessandria et al. (2019).<sup>12</sup>

Figure 13 compares firm-level and aggregate outcomes in this exercise to the first one in which there is no uncertainty. Panels (a) and (b) shows that export participation and market penetration both rise less in response to a risky trade reform in all destinations, although the difference is larger for hard destinations. This is consistent with Handley and Limão (2017)'s finding that U.S.-China trade policy uncertainty depressed Chinese export participation. Panel (c) shows that the effect of trade policy uncertainty on aggregate trade flows is only felt in hard destinations, however. This is because of the high concentration of exports among large firms in easy destinations, whose sales only respond significantly to price-elasticity effects as pointed out by Arkolakis (2010).

Although empirical studies document large negative effects of trade policy uncertainty on export participation and trade flows (Handley and Limão, 2017; Pierce and Schott, 2016), quantitative equilibrium models of sunk-cost-based exporter dynamics typically generate small effects (Steinberg, 2019). These results indicate that market penetration dynamics could account for part of this disconnect, and suggest that uncertainty about trade policy may have larger effects on some trade relationships than others.

<sup>12</sup>The United States unilaterally granted China most-favored-nation status in 1980 but Congress had to renew this status annually. Once China joined the WTO in 2000, Congress's power to revoke this status was eliminated.

## 5.4 Empirical evidence

Emerging economies, particularly those in Latin America, often experience large, persistent episodes of real exchange rate depreciation. Empirical evidence shows that exports tend to grow sluggishly in response to these devaluations and that the source of this sluggishness is adjustment along the extensive margin (Fitzgerald et al., 2016). Brazil’s 1999 depreciation, which exemplifies this pattern, provides a useful way to evaluate the model’s predictions because it can be analyzed using the same customs data from which the facts documented in section 2 were derived. As figure 14 shows, Brazil’s real exchange rate depreciated by more than 50% between 1998 and 1999, and by another 30% by 2003 before beginning to appreciate. Brazil’s aggregate exports, however, grew little until 2002, and the number of exporting firms grew gradually throughout the episode.

In order to study how sales and export participation in different destinations evolved during this period, it is important to control for changes in each destination’s income, price level, and import demand. Argentina, for example, experienced an even larger devaluation than Brazil and a large drop in output when it defaulted on its sovereign debt in 2001. I use the following specification:

$$\log Y_{j,t} = \alpha + \sum_{s=1998}^{2006} \mathbb{1}_{\{t=s\}} \left[ \beta_{s,\text{easy}} \mathbb{1}_{\{\text{group}_j=\text{easy}\}} + \beta_{s,\text{hard}} \mathbb{1}_{\{\text{group}_j=\text{hard}\}} \right] + \delta_1 \log NER_{j,t} + \delta_2 \log CPI_{j,t} + \delta_3 \log RGDP_{j,t} + \delta_4 \log IM_{j,t} + f_j + \epsilon_{j,t}, \quad (37)$$

where the dependent variable  $Y_{j,t}$  is either the total volume of total bilateral trade in U.S. dollars or the number of exporting firms. I split destinations into two groups, “easy” and “hard,” based on their initial levels of export participation in 1998 using the same scheme as in other analyses in this paper. The nominal exchange rate and consumer price index control for changes in each destination’s price level relative to Brazil, while real GDP and imports control for income and multilateral import demand. I also include destination fixed effects to control for the initial level of trade with Brazil in 1998. This specification is in the spirit of a local projection (Jordà, 2005), except that there a single event of interest—the 1999 depreciation—rather than many different events in different destinations and time periods.

Figure 15 plots the coefficient estimates for  $\beta_{s,\text{easy}}$  and  $\beta_{s,\text{hard}}$ . Panel (a) shows that after controlling for destinations’ changes in income, prices, and import demand during this period, exports to hard destinations grew more than exports to easy destinations, and that it took several additional years for exports to the former to reach their peak. Panel (b) shows similar results for export participation. The number of firms exporting to hard destinations grew more than the number of firms exporting to easy destinations, and this growth, too, took longer to come about. Thus, the predictions of the model described in sections 5.1–5.2 are indeed consistent with the evidence from Brazil’s 1999 real exchange rate depreciation.

This evidence is certainly not conclusive given its limited scope. However, it is consistent with findings

reported in several other studies. For example, Mix (2020b) shows that following the formation of free trade area, exports to minor trade partners grow more than exports to major trade partners and that this growth takes longer to materialize. For another, Boehm et al. (2020) shows that when countries lower their most-favored-nation tariff rates, imports from minor trade partners in the WTO grow more than imports from major trade partners. One difference between the model and the data is that trade growth in the short run—periods immediately following a change in prices or trade costs—is larger in the former than the latter. This is a widely known issue that affects all heterogeneous-firm models of trade that are based on the Melitz (2003) framework, not just the model I have developed in this paper. Accounting for the low short-run trade elasticities observed in the data requires additional features like pricing to market (Alessandria and Choi, 2016) and adjustment costs borne by importers Steinberg (2020).

## 6 Conclusion

In this paper, I study how exporting firms' performance dynamics vary across destinations and explore the aggregate implications of these patterns. I use customs data from Brazil to document that in larger, richer markets, new entrants are smaller and more likely to exit relative to incumbents, but successful exporters' sales grow more dramatically over the duration of their export spells. Exporters that serve more than one destination exit from less important ones more frequently, but are more likely to continue exporting to even their lowest-ranked destinations than firms that only sell to one destination.

To account for these facts, I develop a new theory of export market penetration dynamics that synthesizes static models like Arkolakis (2010) and Eaton et al. (2011) in which firms choose how many customers to serve in each destination with dynamic models like Das et al. (2007) and Alessandria and Choi (2007) in which sunk entry costs lead firms to make forward-looking decisions about whether to start and stop exporting. The cost of exporting in the model is increasing in the number of customers a firm wants to serve but decreasing in the number of customers a firm already has, and so firms choose endogenously to grow their customer bases over time. The marginal cost of serving a single customer is strictly positive regardless of the size of a firm's current customer base, which generates endogenous entry and exit. In equilibrium, new exporters pay larger costs relative to the profits they earn from exporting as in models of exogenous new exporter dynamics like Ruhl and Willis (2017) and Alessandria et al. (2014), but these costs vary endogenously across firms and across destinations.

I calibrate the model so that it reproduces a subset of the facts I document in the empirical part of the paper and validate it by demonstrating that it reproduces the remaining facts. I use the calibrated model to explore how aggregate trade dynamics differ across destinations. In response to permanent trade reforms exports to "harder destinations" with smaller, poorer populations grow more than exports to "easier" destinations with larger, richer populations. Moreover, exports to harder destinations exhibit more pronounced hysteresis following temporary real exchange rate depreciations and more sensitivity to uncertainty about

future trade policy. I use the Brazilian customs data to show that these implications are consistent with the bilateral trade dynamics that followed Brazil's 1999 real exchange rate depreciation.

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**Table 1:** Summary statistics of export participation and exporter dynamics across destinations

Statistic	Num. exporters	Top-5 share	Avg. num. dests.	Exit rate	Entrant rel. size	Entrant rel. exit rate
<i>(a) Data</i>						
Mean	576	0.59	16.65	0.40	0.38	0.28
Min	30	0.27	7.77	0.30	0.09	0.07
Max	3,706	0.83	29.34	0.60	0.96	0.38
Std. dev.	819	0.14	5.02	0.07	0.23	0.06
<i>(b) Model</i>						
Mean	966	0.58	18.59	0.44	0.37	0.25
Min	41	0.35	10.53	0.22	0.09	0.07
Max	3,698	0.75	24.67	0.76	1.33	0.42
Std. dev.	875	0.10	3.34	0.13	0.26	0.12

**Table 2:** Associations between destination characteristics and exporter performance

	Log num. exporters	Top-5 share	Avg. num. dests.	Exit rate	Entrant rel. size	Entrant rel. exit rate
<i>(a) Data</i>						
log GDPpc	0.581 (0.023) <sup>§</sup>	0.051 (0.004) <sup>§</sup>	-1.676 (0.155) <sup>§</sup>	-0.006 (0.003) <sup>†</sup>	-0.079 (0.012) <sup>§</sup>	0.010 (0.004) <sup>‡</sup>
log population	0.408 (0.016) <sup>§</sup>	0.047 (0.003) <sup>§</sup>	-1.154 (0.094) <sup>§</sup>	-0.010 (0.002) <sup>§</sup>	-0.044 (0.008) <sup>§</sup>	0.004 (0.003)
log trade barrier	-1.098 (0.029) <sup>§</sup>	-0.065 (0.005) <sup>§</sup>	2.677 (0.199) <sup>§</sup>	0.036 (0.003) <sup>§</sup>	0.096 (0.013) <sup>§</sup>	-0.021 (0.004) <sup>§</sup>
Num. observations	568	568	568	568	568	568
R <sup>2</sup>	0.71	0.43	0.42	0.20	0.13	0.06
<i>(b) Model</i>						
log GDPpc	0.764 (0.003) <sup>§</sup>	0.071 (0.001) <sup>§</sup>	-2.121 (0.006) <sup>§</sup>	-0.073 (0.000) <sup>§</sup>	-0.137 (0.003) <sup>§</sup>	0.036 (0.001) <sup>§</sup>
log population	0.238 (0.002) <sup>§</sup>	0.015 (0.000) <sup>§</sup>	-1.194 (0.004) <sup>§</sup>	0.024 (0.000) <sup>§</sup>	-0.102 (0.003) <sup>§</sup>	0.038 (0.001) <sup>§</sup>
log trade barrier	-0.747 (0.003) <sup>§</sup>	-0.068 (0.001) <sup>§</sup>	2.176 (0.007) <sup>§</sup>	0.070 (0.001) <sup>§</sup>	0.132 (0.003) <sup>§</sup>	-0.038 (0.001) <sup>§</sup>
Num. observations	6,295	6,295	6,295	6,295	6,295	6,295
R <sup>2</sup>	0.97	0.84	0.98	0.91	0.47	0.56

All specifications control for year fixed effects. Robust standard errors in parentheses. §, ‡, and † denote significance at the 0.1%, 1%, and 5% levels, respectively.



**Table 3:** Associations between destination characteristics and average rank in exporters' portfolios

	Average rank
<i>(a) Data</i>	
log GDPpc	-1.280 (0.093) <sup>§</sup>
log population	-1.140 (0.061) <sup>§</sup>
log trade barrier	2.253 (0.124) <sup>§</sup>
Num. observations	568
R <sup>2</sup>	0.55
<i>(b) Model</i>	
log GDPpc	-1.968 (0.008) <sup>§</sup>
log population	-2.490 (0.007) <sup>§</sup>
log trade barrier	2.037 (0.008) <sup>§</sup>
Num. observations	6,295
R <sup>2</sup>	0.98

All specifications control for year fixed effects. Robust standard errors in parentheses. § denotes significance at the 0.1% level.

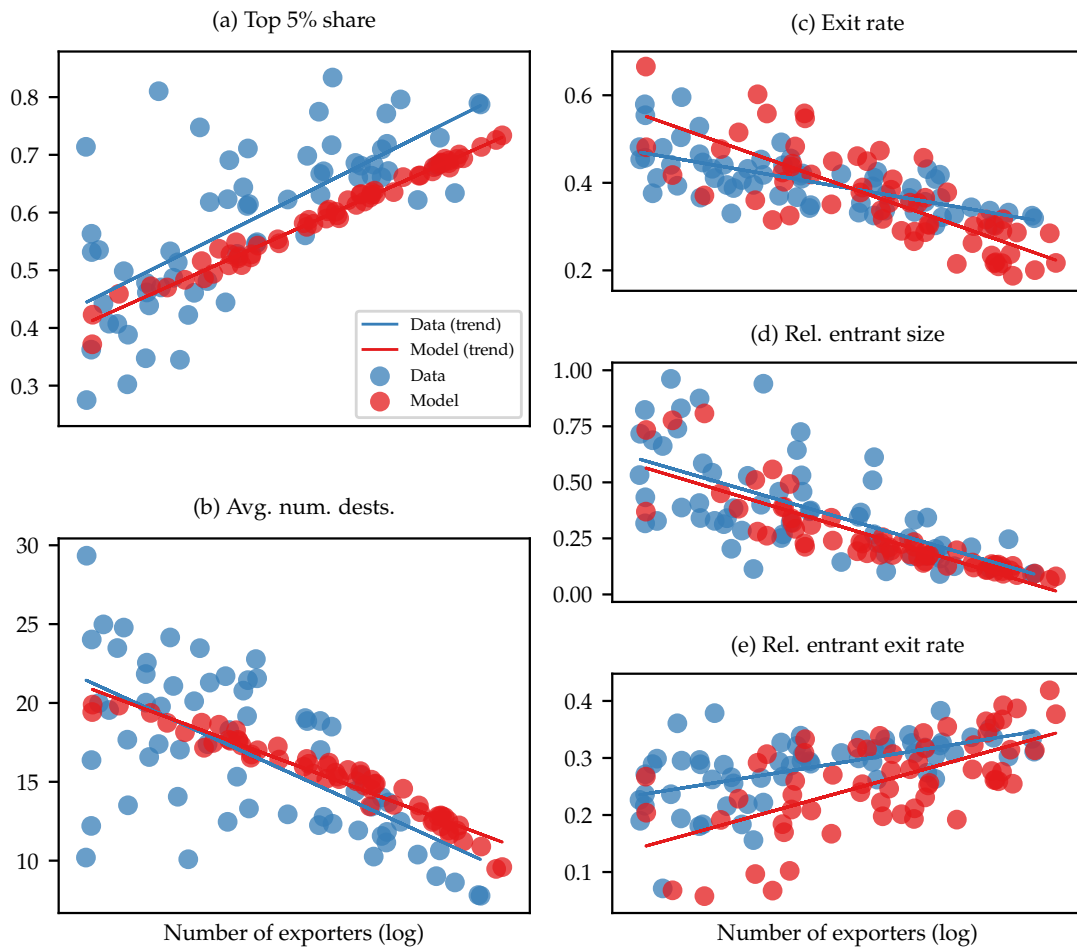
**Table 4:** Exit rates by num. dest. and dest. rank

Num. dest.	Destination rank					
	1	2	3	4	5-9	10+
<i>(a) Data</i>						
1	0.56	-	-	-	-	-
2	0.41	0.60	-	-	-	-
3	0.31	0.47	0.61	-	-	-
4	0.23	0.36	0.49	0.60	-	-
5-9	0.16	0.24	0.32	0.40	0.52	-
10+	0.06	0.08	0.10	0.13	0.21	0.31
<i>(b) Model</i>						
1	0.53	-	-	-	-	-
2	0.39	0.53	-	-	-	-
3	0.30	0.42	0.52	-	-	-
4	0.24	0.35	0.43	0.52	-	-
5-9	0.16	0.24	0.30	0.35	0.42	-
10+	0.05	0.08	0.11	0.13	0.18	0.27

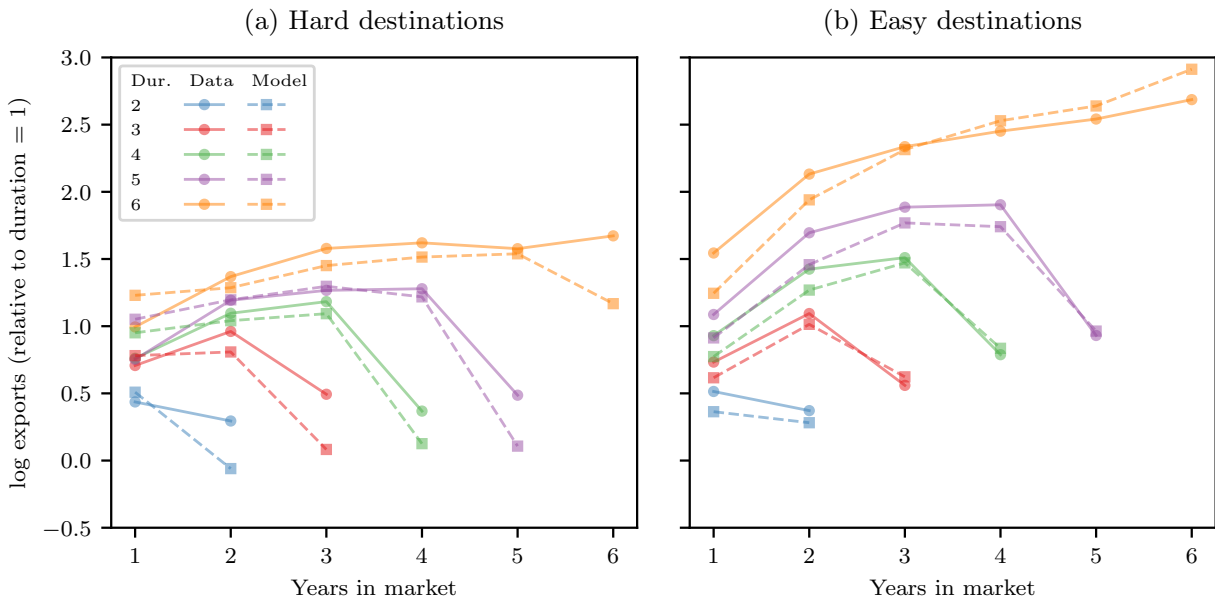
**Table 5: Calibrated parameter values**

Parameter	Meaning	Value
<i>(a) Distribution of firm types</i>		
$\sigma_x$	Variance of productivity	1.02
$\rho_x$	Persistence of productivity	0.98
$\sigma_z$	Variance of demand	0.44
$\rho_z$	Persistence of demand	0.60
$\delta_0$	Correlation of survival with productivity	34.7
$\delta_1$	Minimum death probability	0.03
<i>(c) New customer attraction costs</i>		
$\alpha_n$	Macro return to market size	0.51
$\beta_n$	Micro return to market size	0.94
$\gamma_n$	Convexity	6.50
$\psi_n$	Level	0.10
<i>(d) Old customer retention costs</i>		
$\alpha_o$	Macro return to market size	0.96
$\beta_o$	Micro return to market size	0.79
$\gamma_o$	Convexity	1.75
$\psi_o$	Level	0.06

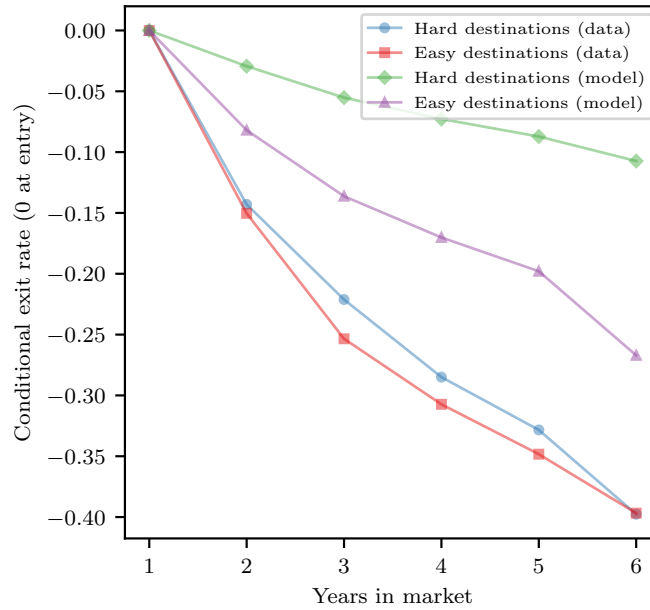
**Figure 1:** Exporter performance vs. export participation



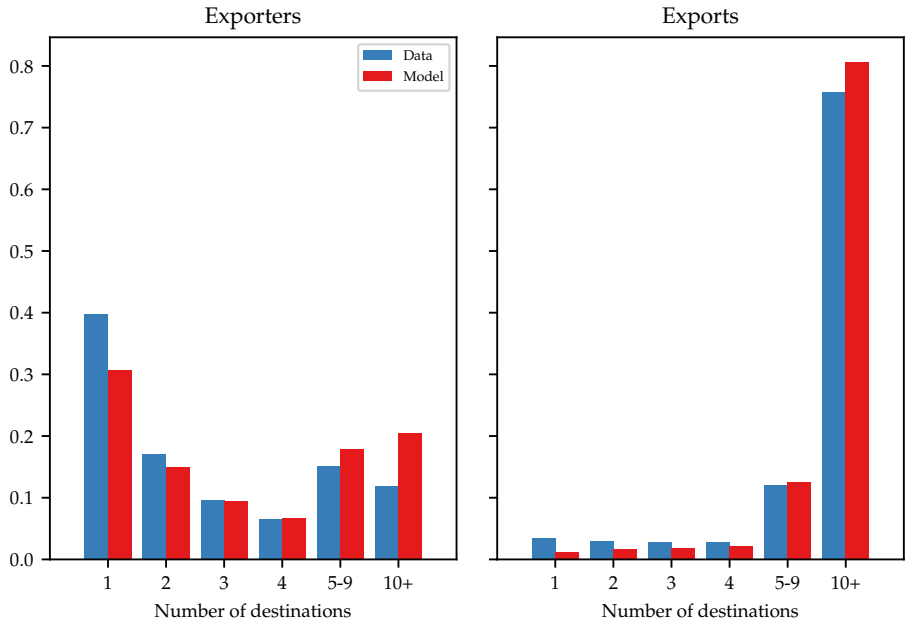
**Figure 2: Effects of tenure and duration on exporters' sales**



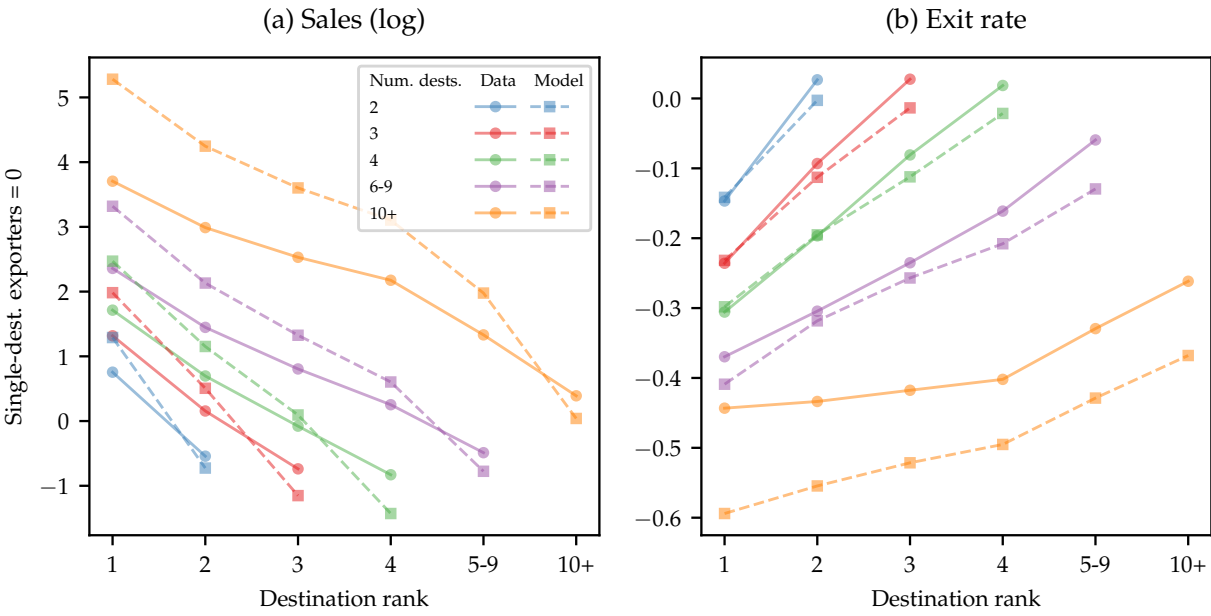
**Figure 3: Exit rates conditional on tenure**



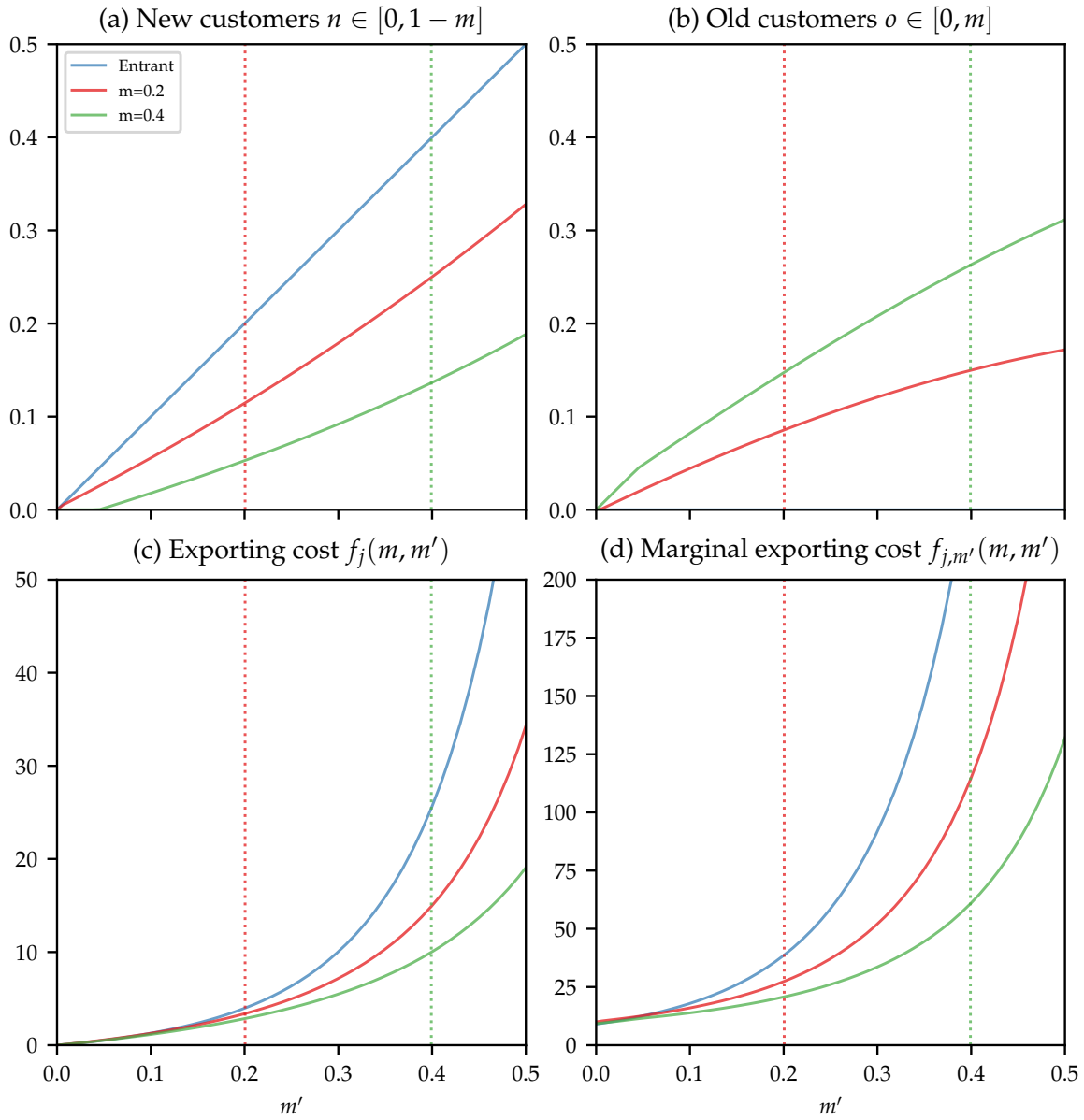
**Figure 4:** Distribution of exporters and exports by number of destinations



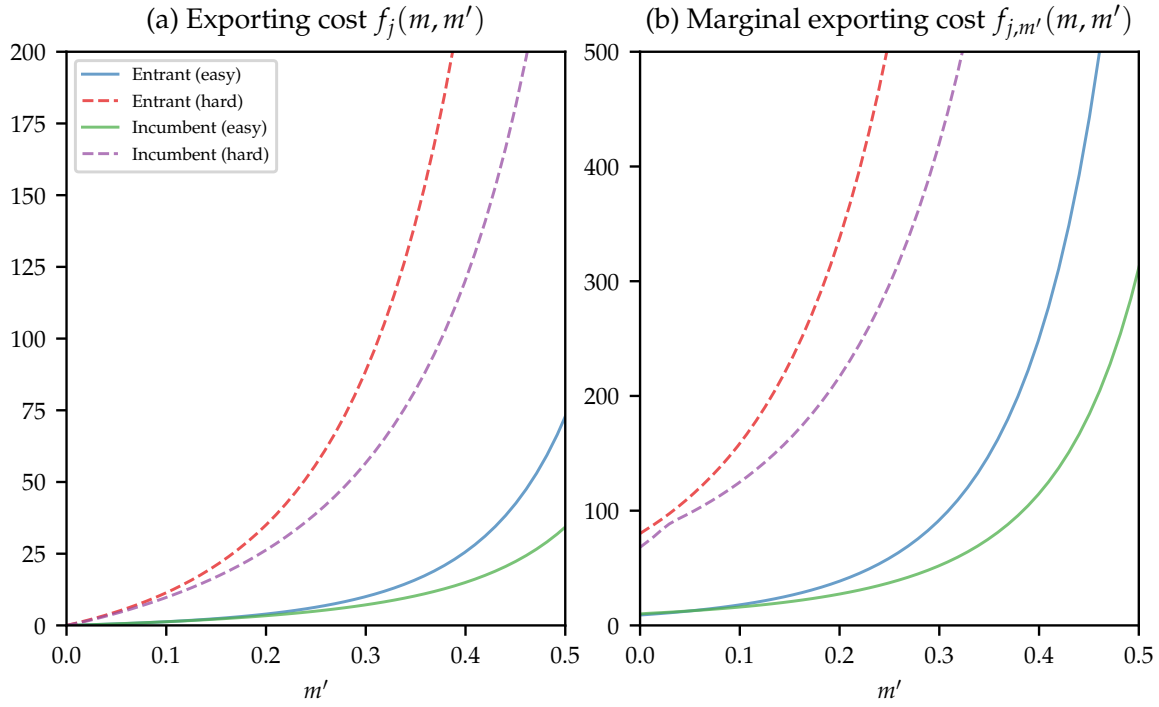
**Figure 5:** Sales and exit rates by number of destinations served and destination rank



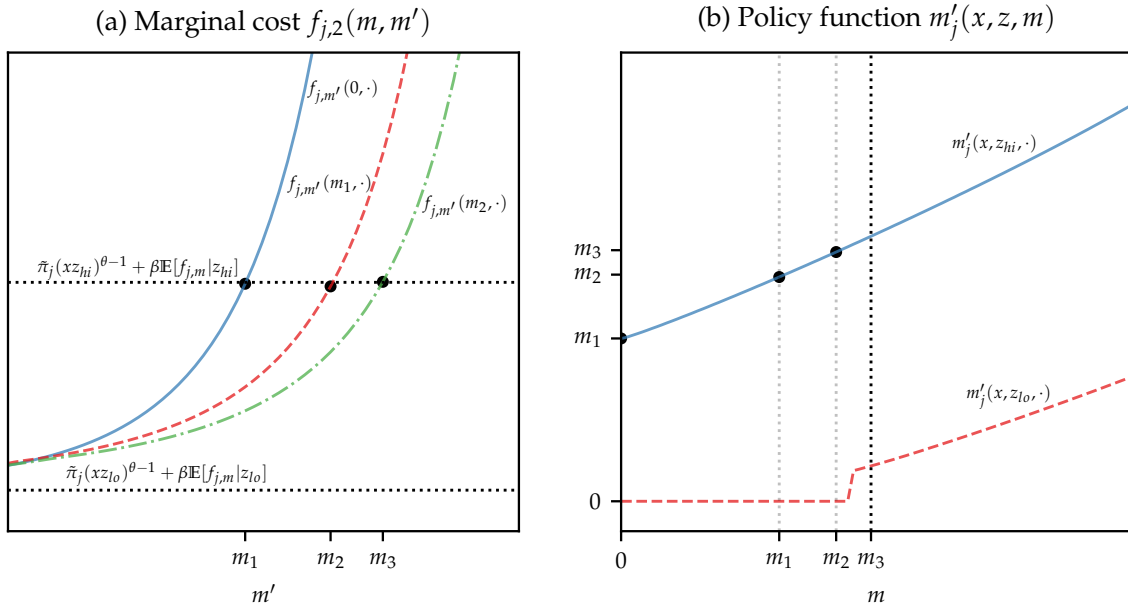
**Figure 6:** Exporting costs and attraction and retention policies



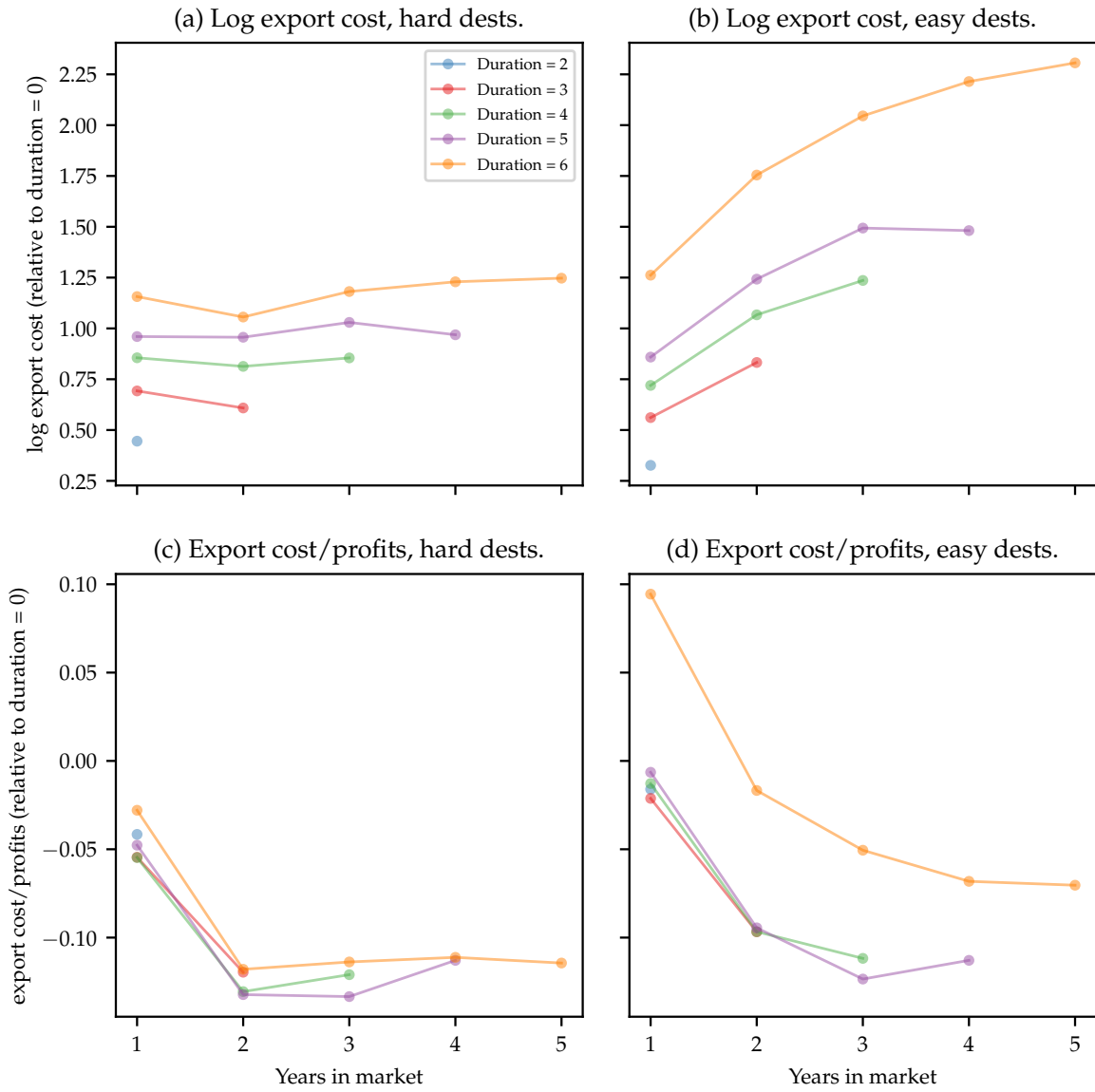
**Figure 7:** Exporting costs in easy vs. hard destinations



**Figure 8:** Illustration of entry, expansion, and exit

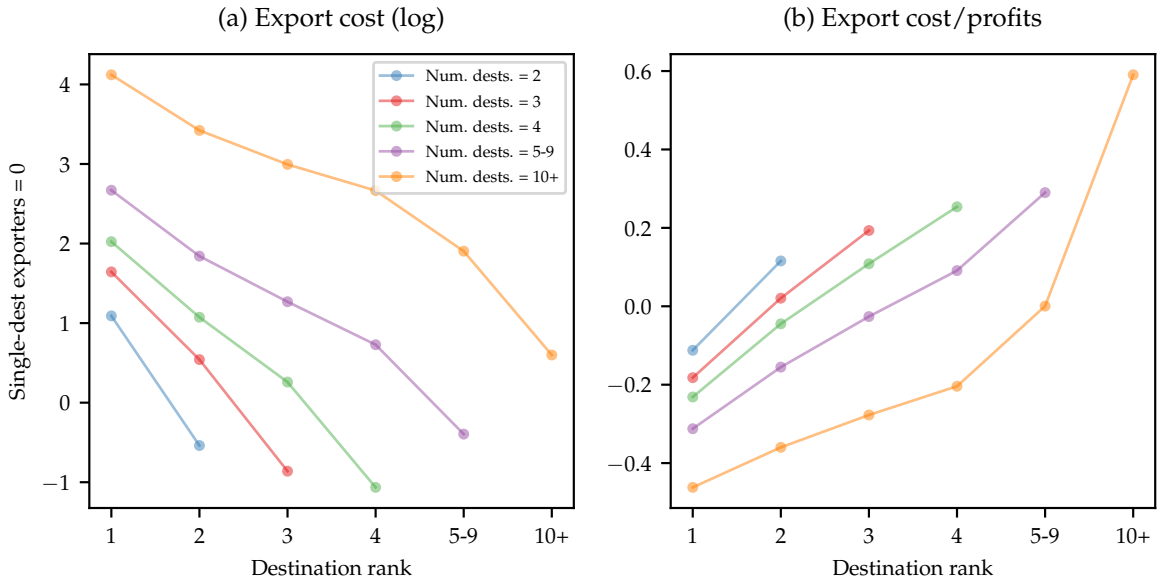


**Figure 9:** Effects of tenure and duration on exporting costs

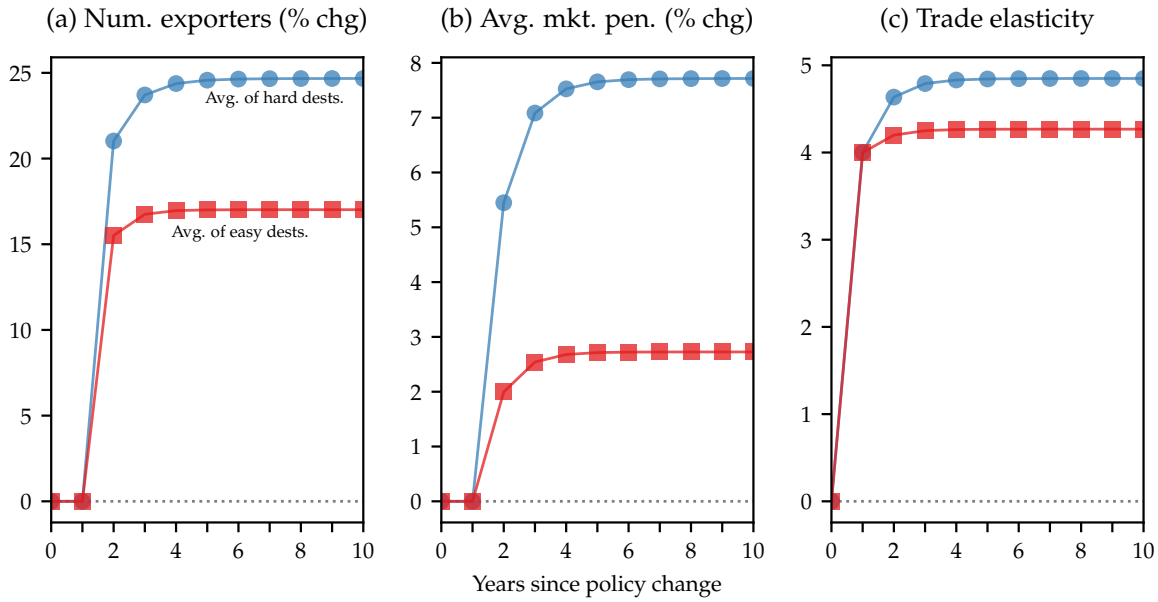




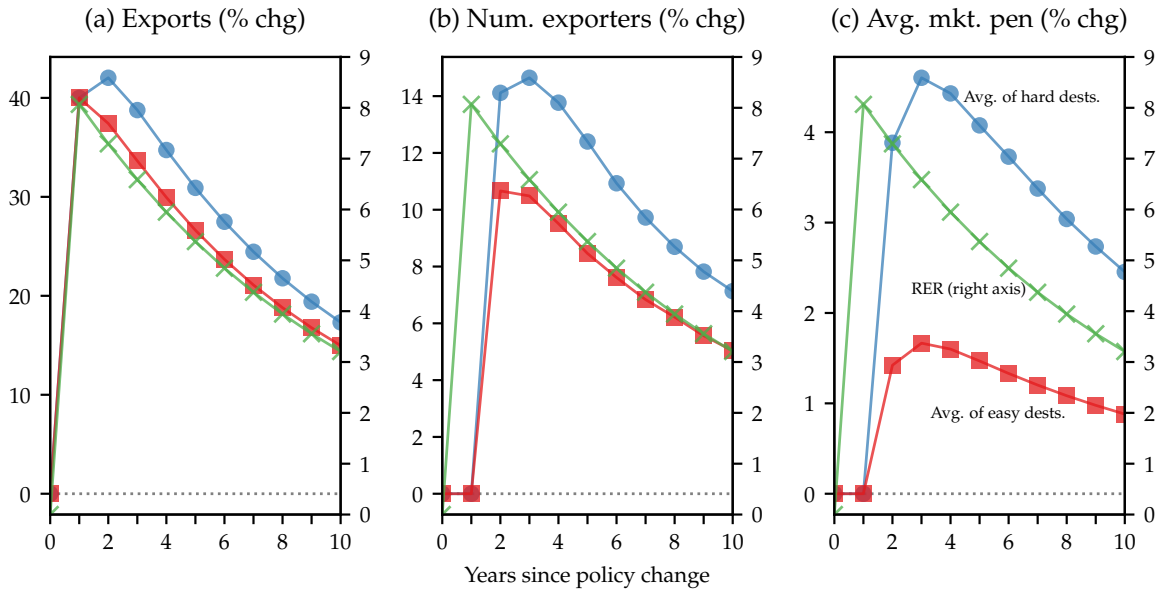
**Figure 10: Exporting costs by number of destinations served and destination rank**



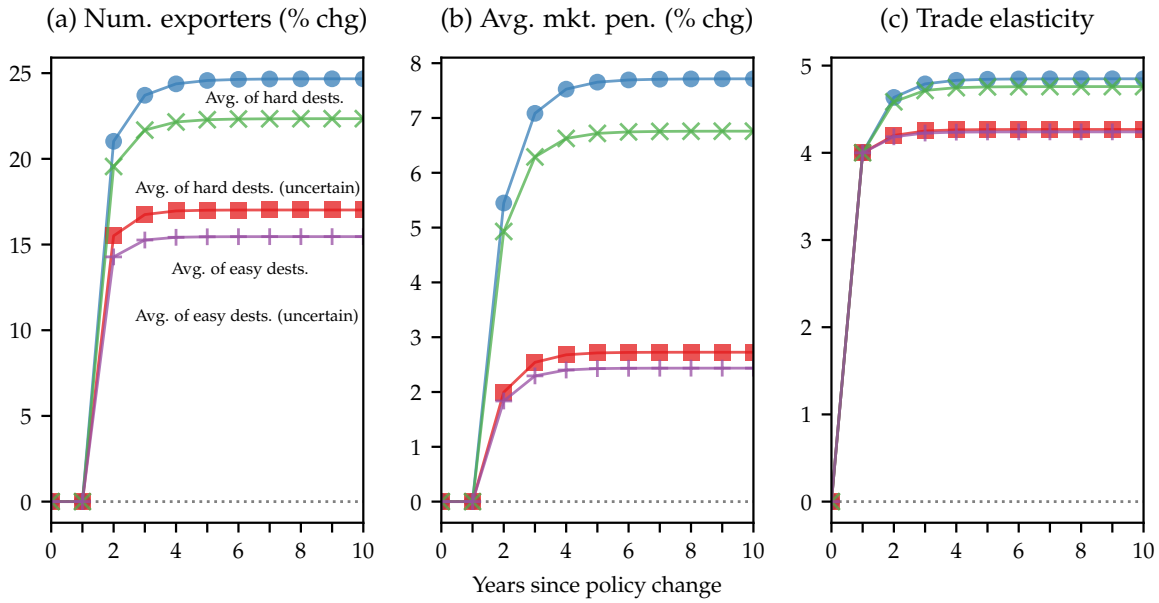
**Figure 11: Transition dynamics following a permanent trade reform**



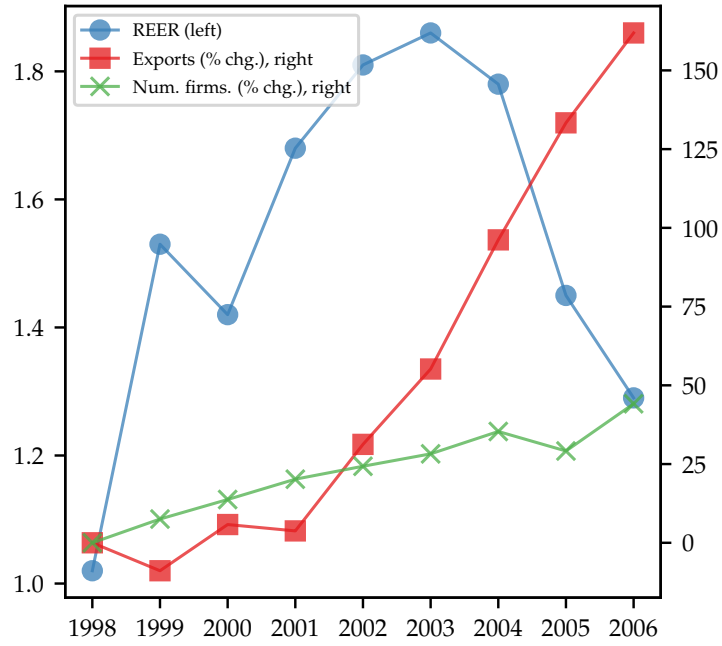
**Figure 12:** Transition dynamics following a temporary real exchange rate depreciation



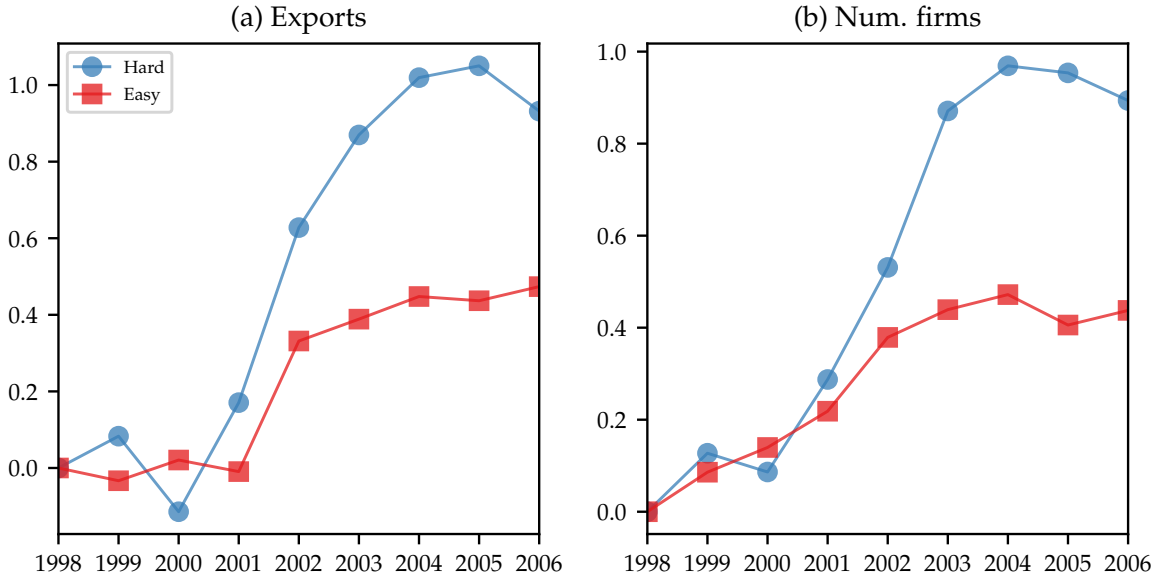
**Figure 13:** Transition dynamics following a trade reform with uncertain duration



**Figure 14: Real exchange rate and trade dynamics in Brazil: 1998–2006**



**Figure 15: Trade and export participation dynamics in hard vs. easy destinations in Brazil: 1998–2006**



## A Empirical results for Mexico and Peru

In this appendix, I report results from empirical analysis of two additional datasets on Mexican and Peruvian exporters from the World Bank's Exporter Dynamics Database. These transaction-level customs datasets have the same structure as the Brazilian data. The Mexican dataset covers the period 2001–2006 and contains about 23,000 firms per year.<sup>13</sup> The Peruvian dataset covers a longer time period, 1994–2008 but contains fewer firms, ranging from 2000 at the beginning of the sample to 5000 at the end. I apply exactly the same processing and analysis procedures described in section 2 to these datasets. Tables A1–A4 and figures A1–A5 show the results.

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<sup>13</sup>The dataset contains information on transactions through 2008, but there is a break in the coding of firm identifiers in 2007.

**Table A1:** Summary statistics of export participation and exporter dynamics across destinations (Mexico and Peru)

Statistic	Num. exporters	Top-5 share	Avg. num. dests.	Exit rate	Entrant rel. size	Entrant rel. exit rate
<i>(a) Mexico</i>						
Mean	661	0.67	15.09	0.46	0.37	0.34
Min	22	0.44	2.22	0.36	0.06	0.16
Max	16,908	0.92	36.50	0.60	1.13	0.54
Std. dev.	2,162	0.12	5.94	0.06	0.27	0.06
<i>(b) Peru</i>						
Mean	133	0.62	10.36	0.48	0.36	0.32
Min	21	0.31	3.63	0.35	0.07	0.16
Max	1,003	0.91	17.61	0.67	0.96	0.43
Std. dev.	178	0.15	3.68	0.07	0.21	0.06

All specifications control for year fixed effects. Robust standard errors in parentheses. §, ‡, and † denote significance at the 0.1%, 1%, and 5% levels, respectively.

**Table A2:** Associations between destination characteristics and exporters' behavior (Mexico and Peru)

	Log num. exporters	Top-5 share	Avg. num. dests.	Exit rate	Entrant rel. size	Entrant rel. exit rate
<i>(a) Mexico</i>						
log GDPpc	0.308 (0.020) <sup>§</sup>	0.065 (0.005) <sup>§</sup>	-1.622 (0.127) <sup>§</sup>	0.012 (0.003) <sup>§</sup>	-0.094 (0.020) <sup>§</sup>	-0.001 (0.004)
log population	0.372 (0.016) <sup>§</sup>	0.040 (0.004) <sup>§</sup>	-1.278 (0.123) <sup>§</sup>	-0.002 (0.002)	-0.048 (0.013) <sup>§</sup>	0.006 (0.004)
log trade barrier	-0.721 (0.015) <sup>§</sup>	-0.031 (0.004) <sup>§</sup>	2.746 (0.123) <sup>§</sup>	0.022 (0.002) <sup>§</sup>	0.084 (0.017) <sup>§</sup>	-0.008 (0.004) <sup>†</sup>
Num. observations	294	294	294	294	294	294
R <sup>2</sup>	0.85	0.51	0.70	0.44	0.12	0.04
<i>(b) Peru</i>						
log GDPpc	0.325 (0.027) <sup>§</sup>	0.077 (0.005) <sup>§</sup>	-0.956 (0.106) <sup>§</sup>	-0.001 (0.003)	-0.062 (0.016) <sup>§</sup>	0.008 (0.005)
log population	0.236 (0.019) <sup>§</sup>	0.042 (0.005) <sup>§</sup>	-0.358 (0.072) <sup>§</sup>	-0.010 (0.003) <sup>§</sup>	-0.058 (0.010) <sup>§</sup>	0.002 (0.004)
log trade barrier	-0.575 (0.026) <sup>§</sup>	-0.059 (0.004) <sup>§</sup>	1.807 (0.096) <sup>§</sup>	0.014 (0.003) <sup>§</sup>	0.099 (0.018) <sup>§</sup>	-0.013 (0.005) <sup>‡</sup>
Num. observations	490	490	490	490	490	490
R <sup>2</sup>	0.64	0.48	0.48	0.12	0.14	0.09

All specifications control for year fixed effects. Robust standard errors in parentheses. §, ‡, and † denote significance at the 0.1%, 1%, and 5% levels, respectively.

**Table A3:** Associations between destination characteristics and average rank within exporters' portfolios (Mexico and Peru)

	Average rank
<i>(a) Mexico</i>	
log GDPpc	-0.974 (0.196) <sup>§</sup>
log population	-1.479 (0.154) <sup>§</sup>
log trade barrier	2.018 (0.175) <sup>§</sup>
Num. observations	454
$R^2$	0.46
<i>(b) Peru</i>	
log GDPpc	-1.414 (0.107) <sup>§</sup>
log population	-0.824 (0.091) <sup>§</sup>
log trade barrier	1.692 (0.119) <sup>§</sup>
Num. observations	894
$R^2$	0.44

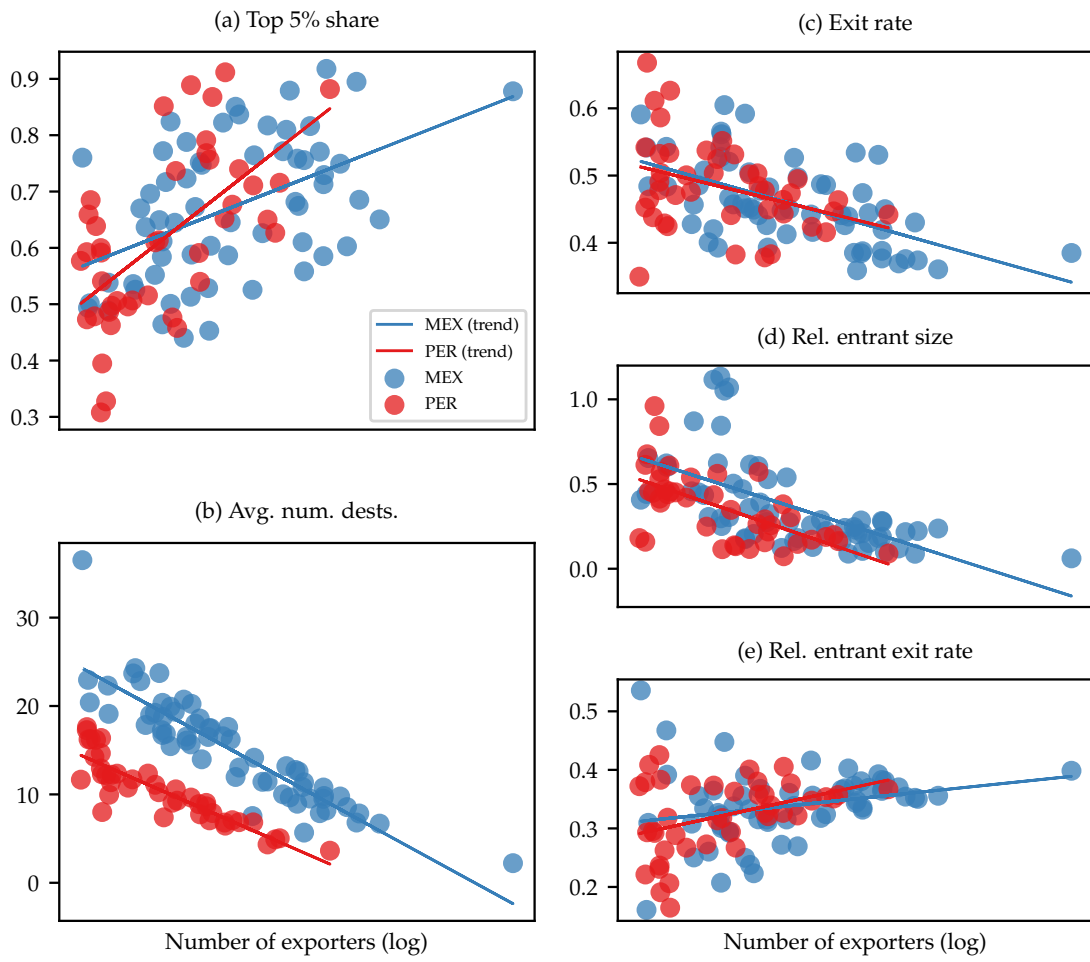
All specifications control for year fixed effects. Robust standard errors in parentheses. §, ‡, and † denote significance at the 0.1%, 1%, and 5% levels, respectively.

**Table A4:** Exit rates by num. dest. and dest. rank (Mexico and Peru)

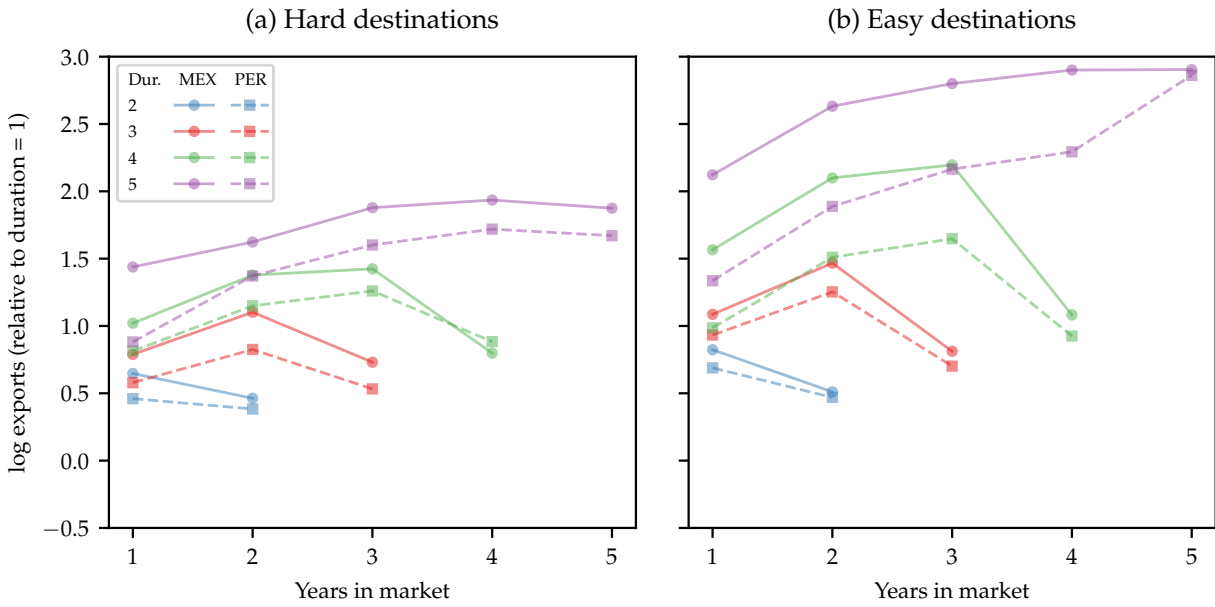
Num. dest.	Destination rank					
	1	2	3	4	5-9	10+
<i>(a) Mexico</i>						
1	0.51	-	-	-	-	-
2	0.36	0.62	-	-	-	-
3	0.27	0.49	0.63	-	-	-
4	0.21	0.39	0.52	0.60	-	-
5-9	0.13	0.26	0.34	0.41	0.49	-
10+	0.05	0.10	0.12	0.14	0.20	0.34
<i>(b) Peru</i>						
1	0.60	-	-	-	-	-
2	0.41	0.60	-	-	-	-
3	0.32	0.50	0.65	-	-	-
4	0.27	0.43	0.55	0.64	-	-
5-9	0.19	0.31	0.40	0.49	0.56	-
10+	0.08	0.12	0.15	0.19	0.26	0.44



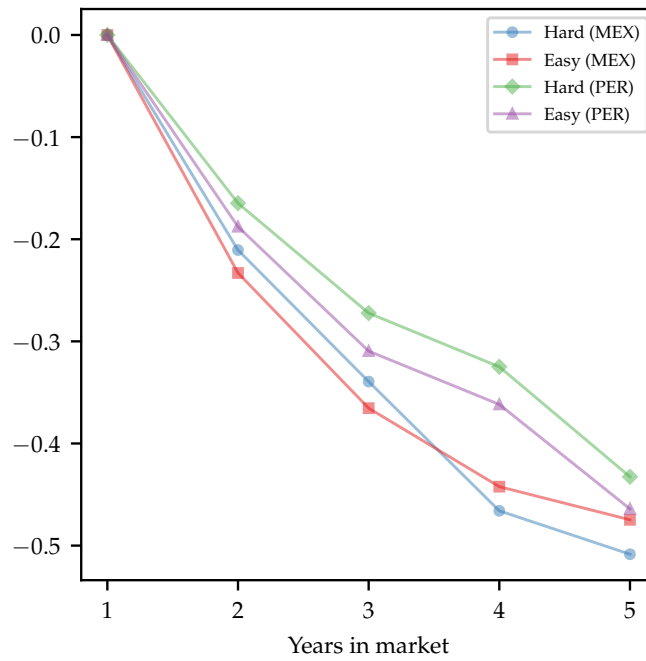
**Figure A1: Exporter performance vs. export participation (Mexico and Peru)**



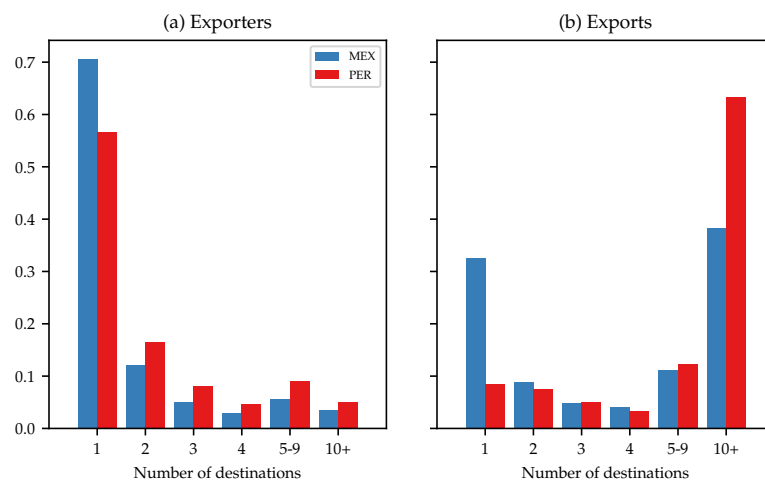
**Figure A2: Effects of tenure and duration on exporters' sales (Mexico and Peru)**



**Figure A3: Exit rates conditional on tenure (Mexico and Peru)**



**Figure A4:** Distribution of exporters and exports by number of destinations (Mexico and Peru)



**Figure A5:** Sales and exit rates by number of destinations served and destination rank (Mexico and Peru)

